STATISTICAL MODEL FOR THE PREDICTION OF SHEAR STRENGTH OF HIGH STRENGTH REINFORCED CONCRETE BEAMS

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1. INTRODUCTION

Shear strength of reinforced concrete (RC) is determined with the help of certain empirical equations based on experimental results from the normal strength reinforced concrete (NSRC) beams.

According to American Concrete Institute Building Code 318 [1], the shear strength of concrete members without transverse reinforcement subject to shear and flexure is given by the following equation:

$$V_c = (0.16\sqrt{f_c'} + 17\rho \frac{V_u d}{M_u}) b_w d \qquad (1)$$

where f_c' = 28 days compressive strength of concrete,

ρ = longitudinal reinforcement in the web,

V_u = factored shear force at the section,

d = effective depth of beam and b_w = web thickness.

$V_c \leq 3.5\ b_w d$ and $\frac{V_u d}{M_u} \leq 1.0$ in computing V_c by Equation (1), where M_u is factored moment occurring simultaneously with V_u at section considered.

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Based on the experimental research on high strength concrete beams, Maphonde and Frantz [2], Sarsam and Al-Musawi [3], and Ahmad *et al*. [4] have shown that Equation (1) overestimates the effect of the compressive strength of concrete and underestimates the effect of shear span to depth ratio on the shear strength of HSRC beams. Hence, Equation (1) is mostly valid for NSRC beams.

Due to lack of test data on compressive strengths of concrete greater than 70 MPa ($\approx$10,000 psi), the 1989 edition of the ACI code imposed a maximum value of 0.70 MPa (100psi), for use in the calculation of shear strength of concrete beams, joists, and slabs. Exceptions to this limit were permitted in beams and joists when the transverse reinforcement satisfied an increased value for the minimum amount of web reinforcement.

Vechhio and Collins [5] developed the Compression Field Theory (CFT) to study the effect of tensile stresses on the shear strength of RC beams in the cracked region. The nominal shear capacity V_n of the reinforced concrete section is given as:

$$V_n = V_s + V_p \tag{2}$$

V_c = shear strength provided by the cracked concrete

V_s = shear strength provided tensile stress in stirrups

V_p = vertical component of applied pre-stressed tendons

$$V_n = \beta\sqrt{f_c'}\,b_w d_w + A_v f_v Cot\theta + V_p \tag{3}$$

β = concrete tensile stress factor indicating the ability of diagonally cracked concrete to resist shear $d_w \approx 0.9\, d$ = the minimum web depth Rameriz and Breen [6] suggested the following model for nominal shear strength of concrete beams without web reinforcement:

$$V_n = 0.5(3V_{cr} - \theta)b_w d \tag{4}$$

V_{cr} = shear stress resulting in the first diagonal tension cracking in the concrete θ = crack angle in radians

Gambarova [7] and Dei Poli *et al.* [8] developed the approach of the truss model, which is based on the assumption that the forces are transferred across the crack by the friction which depends on the crack displacement (slip and crack width). They proposed the following equation for the contribution of web reinforcement in resisting the shear in RC beams:

$$V_s = \frac{A_v f_y Cot\beta_{cr}}{s} \tag{5}$$

where β_{cr} = crack inclination d_v = inner lever arm s = stirrup spacing Karim *et al*. [9] proposed the following equation for predication of ultimate shear stress in beams without web reinforcement.

$$v_c = \frac{V_u}{bd} = 0.4 + \sqrt{\frac{f_c'(\rho d)}{a}(10 - 3A_d)} \qquad \text{(SI Units)} \tag{6}$$

where $A_d = \frac{a}{d}$ for $1.0 < \frac{a}{d} < 2.5$ and 2.5 for $\frac{a}{d} \geq 2.5$

Zararis [10] has proposed the following models for the shear strength of beams without web reinforcement:

$$V_{cr} = \left[1.2 - 0.2(\frac{a}{d})d\right]\left(\frac{c}{d}\right)f_{ct}bd \tag{7}$$

where $\left[1.2 - 0.2\left(\frac{a}{d}\right)d\right] \geq 0.65$ (d in meters)

$$f_{ct} = 0.3\left(f_c'\right)^{\frac{2}{3}}$$

c is depth of compression zone which is determined by the following quadratic equation:

$$\left(\frac{c}{d}\right)^2+\frac{600(\rho+\rho')}{f_c'}\left(\frac{c}{d}\right)-\frac{600}{f_c'}\left[\rho+\left(\frac{d}{d'}\right)\rho'\right]=0 \qquad \text{(SI Units)} \tag{8}$$

For beams with shear reinforcement, the steel contribution is added, which is expressed as

$$V_s=\left[0.5+0.25\left(\frac{a}{d}\right)\right]\rho_y f_{vy} bd \qquad \text{(SI units)} \tag{9}$$

The shear strength of HSRC beams is still determined by using the equations of Normal Strength Reinforced Concrete (NSRC) beams by most of the codes. However, various researchers have proposed certain empirical equations for shear strength of HSRC beams on the basis of test results and mathematical models.

Sarkar *et al*. [11] worked on reinforced concrete beams with compressive strength ranging from 40 MPa to 110 MPa. They proposed the following two equations for the shear stress of reinforced concrete beams without web reinforcement.

For beams having a/d $\leq$ 2 $\qquad v_c=4.13\left(f_c'.\rho.d\right)^{0.66}$

For beams having a/d > 2 $\qquad v_c=3.05\left(f_c'.\rho.d\right)^{0.55}$ (SI Units) (10)

Bazant and Kim [12] proposed a very reliable expression for computing the shear strength of RC beams without transverse reinforcement, which is given as

$$v_{uv}=\zeta\left[0.83\rho^{1/3}f_c'^{1/2}+206.9\rho^{5/6}\left(\frac{a}{d}\right)^{-5/2}\right] \qquad \text{(SI Units)} \tag{11}$$

Where $\zeta=1/\sqrt{a+\frac{d}{25d_a}}$ is a function taking into account the size effect of aggregates, and where d_a stands for aggregates sizes.

Russo *et al*. [13] proposed the following expression for shear strength of HSRC concrete beams without transverse reinforcement based on Bazant and Kim's equation:

$$v_{uv}=\zeta\left[0.97\rho^{0.46}f_c'^{1/2}+0.2\rho^{0.91}f_c'^{0.38}f_{yl}^{\ 0.95}\left(\frac{a}{d}\right)^{-2.33}\right] \tag{12}$$

They further proposed the following expression for the shear strength of HSRC beams with transverse reinforcement using the above expression:

$$v_{uv}=\zeta\left[0.97\rho^{0.46}f_c'^{1/2}+0.2\rho^{0.91}f_c'^{0.38}f_{yl}^{\ 0.95}\left(\frac{a}{d}\right)^{-2.33}\right]+1.75I_b\rho_v f_{yv} \tag{13}$$

The factor I_b is given by the equation

$$I_b=\frac{0.97\rho^{0.46}f_c'^{1/2}}{0.97\rho^{0.46}f_c'^{1/2}+0.2\rho^{0.91}f_c'^{0.38}f_{yl}^{\ 0.95}\left(\frac{a}{d}\right)^{-2.33}} \tag{14}$$

To check whether the shear failure is due to beam action or arch action, the author further proposed a critical value as

$$\left(a/d\right)_c=0.57\frac{\rho^{0.19}f_{yl}^{\ 1/2}}{fc'^{0.05}} \qquad \text{(SI units)} \tag{15}$$

Hence I_{bc} = 0.57, which means that for i). $a/d < (a/d)_c$ $I_b < 0.57$, arch action prevails

ii). $a/d > (a/d)_c$ $I_b > 0.57$, beam action prevails

Cladera and Mari [14,15] proposed the following equations for the shear strength of beams without web reinforcement

$$V_c = \left[0.225\zeta(100\rho)^{1/2} f_c'^{0.2}\right] b_w d \quad \text{(SI Units)} \tag{16}$$

For beams with web reinforcement, the shear strength is given as

$$V_c = \left[0.17\xi(100\rho)^{1/2} f_c'^{0.20} \tau^{1/3}\right] b_w d \tag{17}$$

$$V_s = \frac{d_v A_w}{s(f_{yw} d \cot\theta)} . \tag{18}$$

In the present research, seventy high-strength concrete-reinforced (HSRC) beams with and without web shear have been tested under monotonic load at mid span. Based on the test results, two regression equations have been proposed for predicting the shear strength of HSRC beams. The results have been compared with the existing models proposed by different researchers.

2. DETAILS OF MATERIAL AND TEST SAMPLES

To study the behavior of high strength concrete beams in shear, with and without shear reinforcement, seventy beams in two series of thirty-five beams each of size 23cm x 300 cm (9 in x 12 in) were prepared. Seven values of shear span to depth (a/d) ratios were used to study mainly the slender beams (shear span to depth ratio a/d were taken as 3, 3.5, 4, 4.5, 5, 5.5, 6). For each value of a/d, five types of longitudinal steel ratios were used (ρ = 0.0033, 0.0075, 0.01, 0.015, 0.02) to study the effect of longitudinal steel ratios on the shear strength of HSRC beams.

For series-I, thirty-five beams were used without transverse reinforcement, whereas in series-II, thirty-five beams having shear reinforcement with #2 bars @ 6″ c/c were used, which corresponds to minimum shear reinforcement as per ACI-318 code provisions.

2.1. Material

2.1.1. Reinforcing steel

For main reinforcement, deformed steel bars having nominal yield stress of 414 MPa (60,000 psi) have been used. For shear reinforcement, plain steel bars of yield stress 276 MPa (40,000 psi) were used.

2.1.2. Concrete

The concrete mix design of the beams used in this experimental program has been given in Table 1. Coarse aggregates of size ¾ in (20mm) and fine aggregates conforming to ASTM standards with modulus of fineness as 2.67 were used in the concrete. High range water reducers conforming to ASTM C-494 type F standards was used at 1.70 % by weight of cement to control the water cement ratio and enhance the compressive strength of concrete. The details of beam sizes, main reinforcement, shear reinforcement, and a/d ratios are shown in Table 2 and a typical section of the beams is given in Figure 1. The loading arrangements are shown in Figure 2.

Table 1. Mix Proportioning/ Designing of High Strength Concrete

Constituent	Proportion
Type- I Cement	628 kg/m^3
Fine aggregates	484 kg/m^3
Coarse aggregates	1128 kg/m^3
HRWR @ by weight of cement	10.70 kg/m^3
Water @ 0.25 w/c ratio	157 kg/m^3
Average Design Cylinder Compressive strength (28 days) f_c'	50-54 MPa

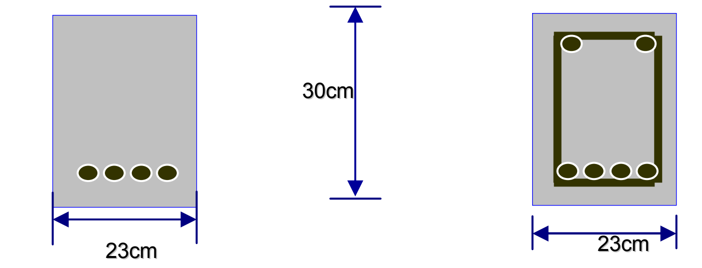


Figure 1. Typical section of beams without and with stirrups

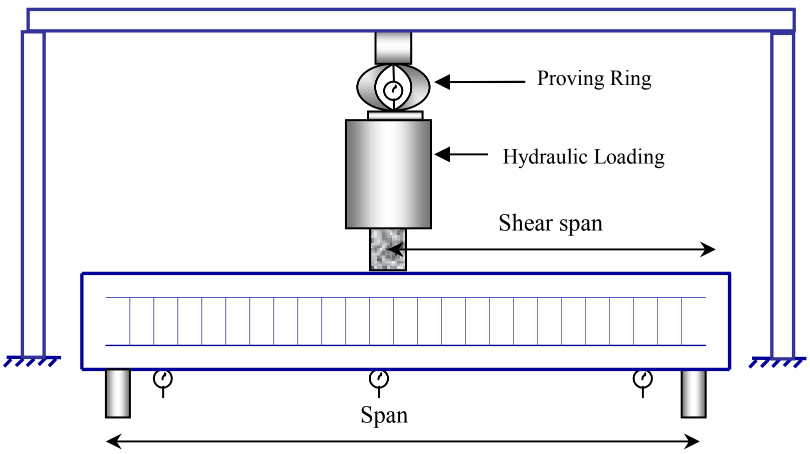


Figure 2. Typical loading arrangement for testing of beams

Table 2. Details of Beams With and Without Shear Reinforcement

Beams without stirrups Series-I				Beams with stirrups Series-II				
Beam	ρ (%)	a/d	Span (cm)	Beam	ρ (%)	a/d	Span (cm)	Stirrups ρ_v(%)
B1-1	0.33	3.0	152.40	Bs1-1	0.33	3.0	152.40	0.1
B1-2	0.33	3.5	177.80	Bs1-2	0.33	3.5	177.80	0.1
B1-3	0.33	4.0	203.20	Bs1-3	0.33	4.0	203.20	0.1
B1-4	0.33	4.5	228.60	Bs1-4	0.33	4.5	228.60	0.1
B1-5	0.33	5.0	254.00	Bs1-5	0.33	5.0	254.00	0.1
B1-6	0.33	5.5	279.40	Bs1-6	0.33	5.5	279.40	0.1
B1-7	0.33	6.0	304.80	Bs1-7	0.33	6.0	304.80	0.1
B2-1	0.73	3.0	152.40	Bs2-1	0.73	3.0	152.40	0.1
B2-2	0.73	3.5	177.80	Bs2-2	0.73	3.5	177.80	0.1
B2-3	0.73	4.0	203.20	Bs2-3	0.73	4.0	203.20	0.1
B2-4	0.73	4.5	228.60	Bs2-4	0.73	4.5	228.60	0.1
B2-5	0.73	5.0	254.00	Bs2-5	0.73	5.0	254.00	0.1
B2-6	0.73	5.5	279.40	Bs2-6	0.73	5.5	279.40	0.1
B2-7	0.73	6.0	304.80	Bs2-7	0.73	6.0	304.80	0.1
B3-1	1.00	3.0	152.40	Bs3-1	1.00	3.0	152.40	0.1
B3-2	1.00	3.5	177.80	Bs3-2	1.00	3.5	177.80	0.1
B3-3	1.00	4.0	203.20	Bs3-3	1.00	4.0	203.20	0.1
B3-4	1.00	4.5	228.60	Bs3-4	1.00	4.5	228.60	0.1
B3-5	1.00	5.0	254.00	Bs3-5	1.00	5.0	254.00	0.1
B3-6	1.00	5.5	279.40	B3-6	1.00	5.5	279.40	0.1
B3-7	1.00	6.0	304.80	Bs3-7	1.00	6.0	304.80	0.1
B4-1	1.50	3.0	152.40	Bs4-1	1.50	3.0	152.40	0.1
B4-2	1.50	3.5	177.80	B4-2	1.50	3.5	177.80	0.1
B4-3	1.50	4.0	203.20	Bs4-3	1.50	4.0	203.20	0.1
B4-4	1.50	4.5	228.60	Bs4-4	1.50	4.5	228.60	0.1
B4-5	1.50	5.0	254.00	Bs4-5	1.50	5.0	254.00	0.1
B4-6	1.50	5.5	279.40	Bs4-6	1.50	5.5	279.40	0.1
B4-7	1.50	6.0	304.80	Bs4-7	1.50	6.0	304.80	0.1
B5-1	2.0	3.0	152.40	Bs5-1	2.0	3.0	152.40	0.1
B5-2	2.0	3.5	177.80	Bs5-2	2.0	3.5	177.80	0.1
B5-3	2.0	4.0	203.20	Bs5-3	2.0	4.0	203.20	0.1
B5-4	2.0	4.5	228.60	Bs5-4	2.0	4.5	228.60	0.1
B5-5	2.0	5.0	254.00	Bs5-5	2.0	5.0	254.00	0.1
B5-6	2.0	5.5	279.40	Bs5-6	2.0	5.5	279.40	0.1
B5-7	2.0	6.0	304.80	Bs5-7	2.0	6.0	304.80	0.1

3. TESTING AND OBSERVATIONS

The beams were tested under monotonic concentrated load at the mid span. The schematic testing arrangements of the beams is shown in Figure 2. The beams were cured with moist sand in the curing yard as shown in Figure 3.

When the load was applied and increased gradually, flexural cracks appeared in the beams near the mid span of the beams, which were more or less vertical in nature. With further increase of load, inclined shear cracks developed in the beams, which are sometimes called primary shear cracks as well.

Typical cracking in the slender beams without transverse reinforcement leading to the failure involved two branches. The first branch was a slightly inclined shear crack and was typically of the height of the other flexural cracks developed on the surface of beams. The second branch of the crack, also called secondary shear crack, initiated from the tip of the first crack at a relatively flatter angle, splitting the concrete in the compression zone. This crack further extended in the compression zone and finally met the loading point, leading to the collapse of the beam as shown in Figure 4. The nominal shear strength at the initiation of the second branch crack was taken as the shear capacity of the beams. In case of beams without transverse reinforcement, the secondary shear crack formed shortly after the development of the primary shear crack and the shear failure was sudden, as shown in Figure 5

In the case of beams with transverse reinforcement, the formation of secondary shear crack was not abrupt and beams carried more loads before failure as compared to beams without web reinforcement. In both cases, the shear strength of the beams was taken at the point when secondary shear cracks appeared.

Figure 3. Beams are stacked in the testing yard after casting and curing

Figure 4. The formation of secondary cracks originating from the primary cracks leading to failure of beams without web reinforcing

Figure 5. Typical shear failure of beam without web reinforcement. The failure is more brittle and sudden. Brick has been placed under the beam to avoid its splitting in two parts (ρ = 1.5 % , a/d = 5.5 span = 321 cm).

4. REGRESSION MODEL FOR SHEAR STRENGTH OF BEAMS WITHOUT WEB REINFORCEMENT

The shear strength of HSRC beams was investigated as a function of longitudinal steel ρ expressed in percentage (%), shear span to depth ratio (*a/d*), compressive strength of concrete, and f_c' (Mpa) for beams without shear reinforcement. The Ordinary Least Square (OLS) estimate for the linear regression model is given as

$$V_c = \left[0.026 f_c' + 0.507\rho - 0.208(a/d)\right]bd \tag{19}$$

The actual and predicted values of shear stress based on the linear regression model have been compared in Figure 6 and Figure 7.

5. LINEAR REGRESSION MODEL FOR SHEAR STRENGTH OF BEAMS WITH WEB REINFORCEMENT

According to ACI-318, the shear contribution of stirrups in beams with web reinforcement is given as

$$V_s = \frac{A_v f_y}{s} d \tag{20}$$

According to ACI-318, the nominal shear strength of RC beams with web reinforcement is the sum of the individual contributions of concrete and steel.

$$V_n = V_c + V_s = (\nu_c + \nu_s) b_v d \tag{21}$$

However, research by Sarsam and Al-Musawi [16] and P. Regan [17] has revealed that the behavior of stirrups in resisting the shear of RC beams is more complicated and irregular. The actual results of 35 beams with web reinforcement have also shown that the increase in shear strength due to stirrups exhibits a non-linear and non-uniform behavior in resisting the shear.

The linear regression model o was worked out for the data of 35 beams tested in the research and the following equation was obtained:

$$V_c = \left[0.01 f_c' + 0.507\rho - 0.208(a/d) + 4.53\rho_v f_y\right]bd \tag{22}$$

The actual and predicted values by the models have been compared in Figure 8 and Figure 9.

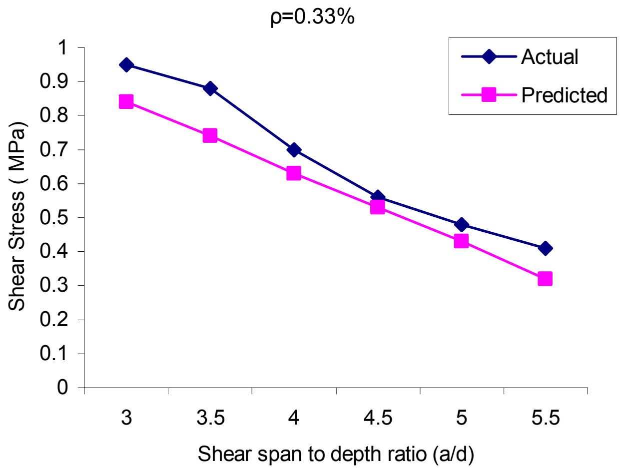
6. COMPARISON OF THE PROPOSED MODELS WITH ACI-318 CODE AND OTHER MODELS

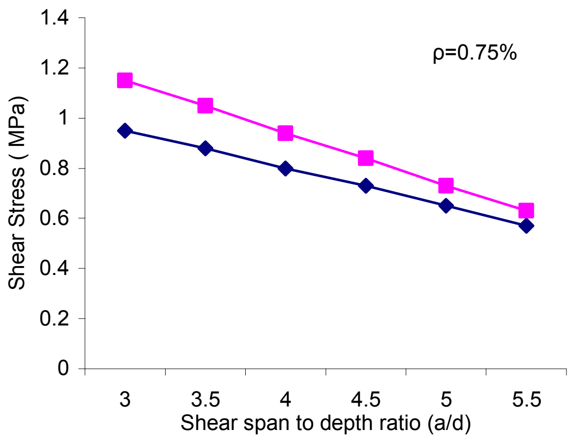
6.1. Beams without Shear Reinforcement

The shear strength of beams without shear reinforcement was compared with the ACI equations [1] and other models proposed by Bezant and Kim [12] and G. Russo *.et.al.* [13]. The comparison has been given in Figure 10.

6.2. Beams with Shear Reinforcement

The predicted results of shear strength for beams with shear reinforcement has been compared with the results of ACI Equation [1] and model proposed by G. Russo *et al*. [13]. The comparison has been given in Figure 11.





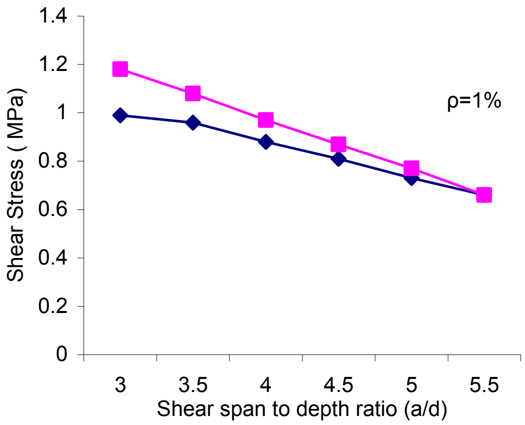


Figure 6. Comparison of actual and predicted values of shear stress in beams without web reinforcement for ρ≤1%

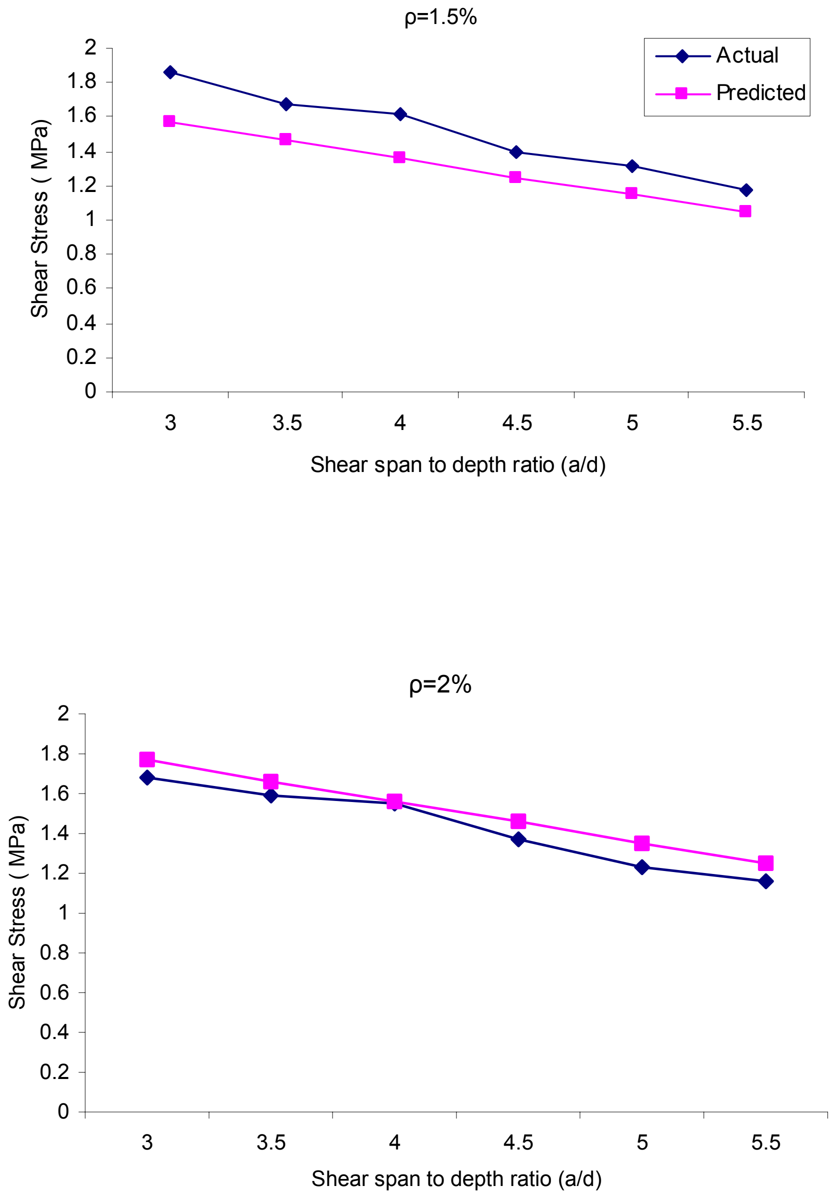


Figure 7. Comparison of actual and predicted values of shear stress in beams without web reinforcement for ρ>1%

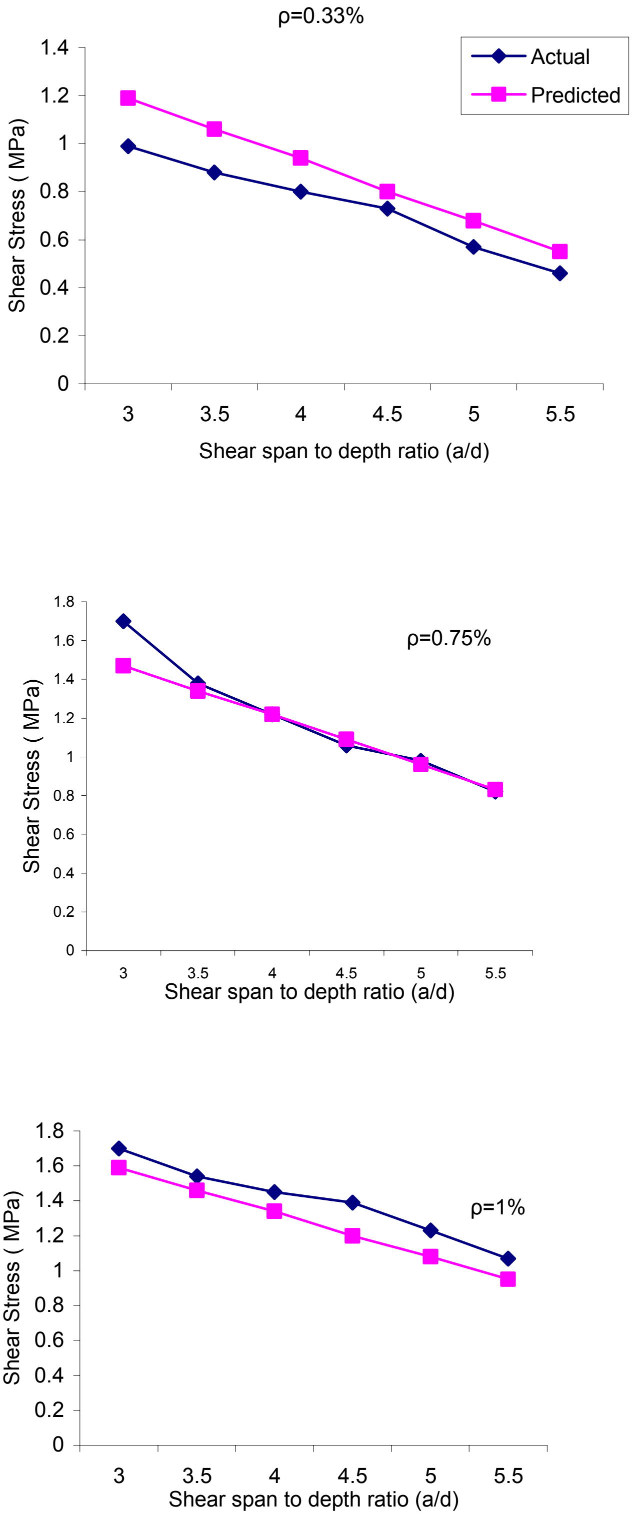


Figure 8. Comparison of actual and predicted values of shear stress in beams with web reinforcement for ρ≤1%

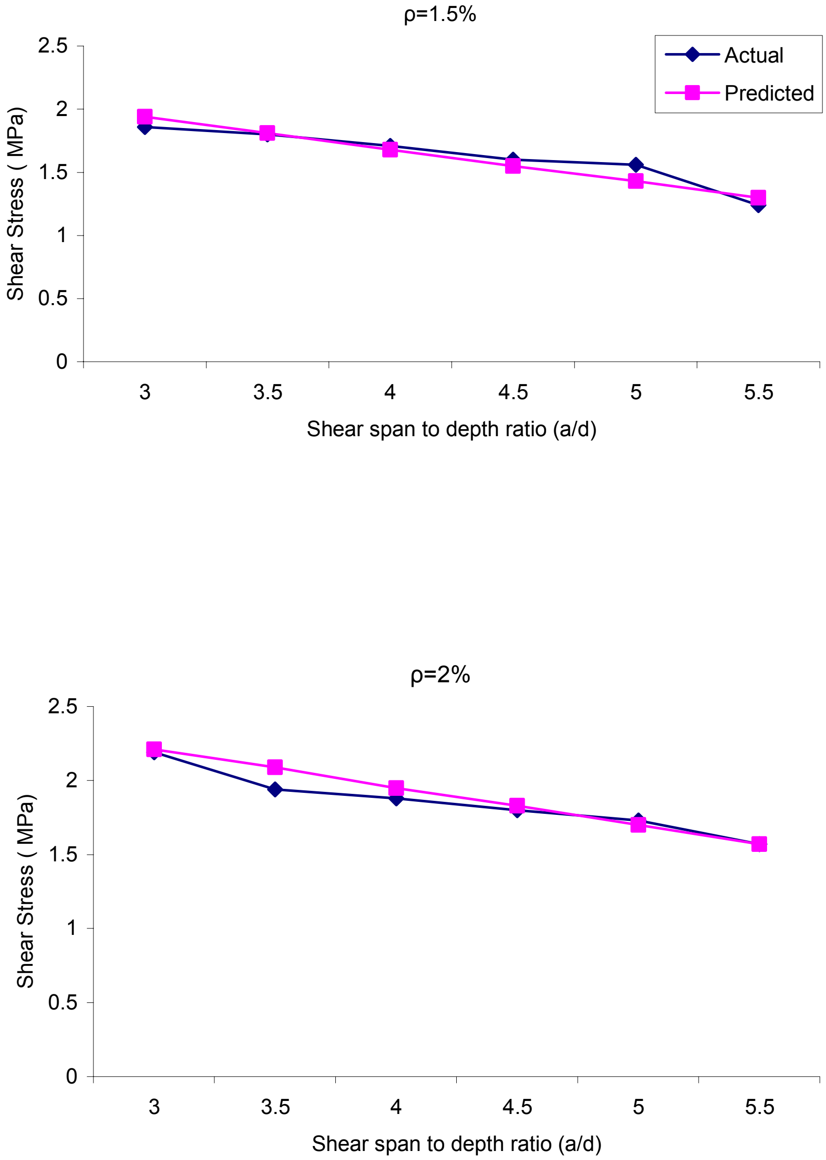


Figure 9. Comparison of actual and predicted values of shear stress in beams with web reinforcement for ρ>1%

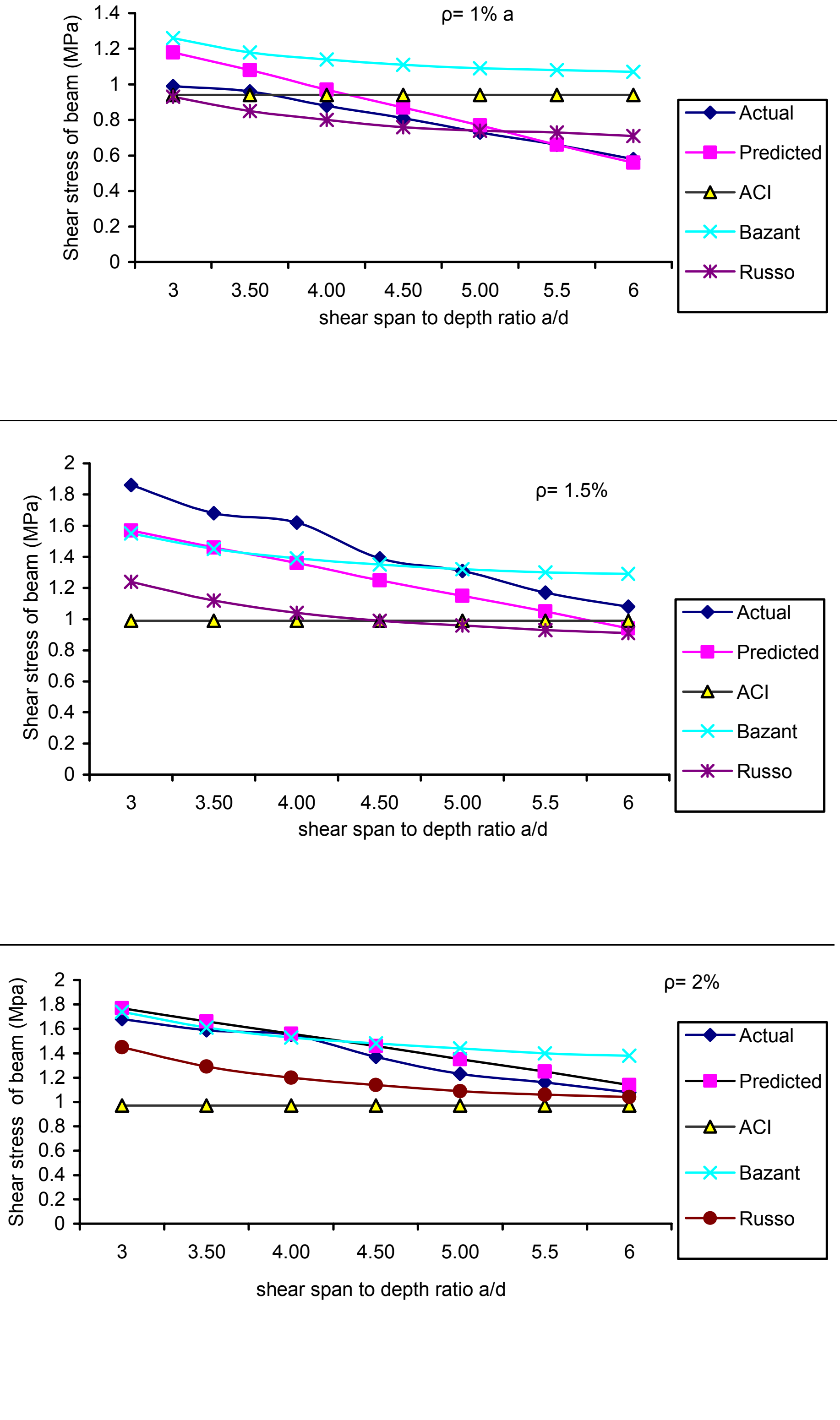


Figure 10. Comparison of actual shear stress of beams having no shear reinforcement with the proposed equation and other models for $\rho \geq 1\%$

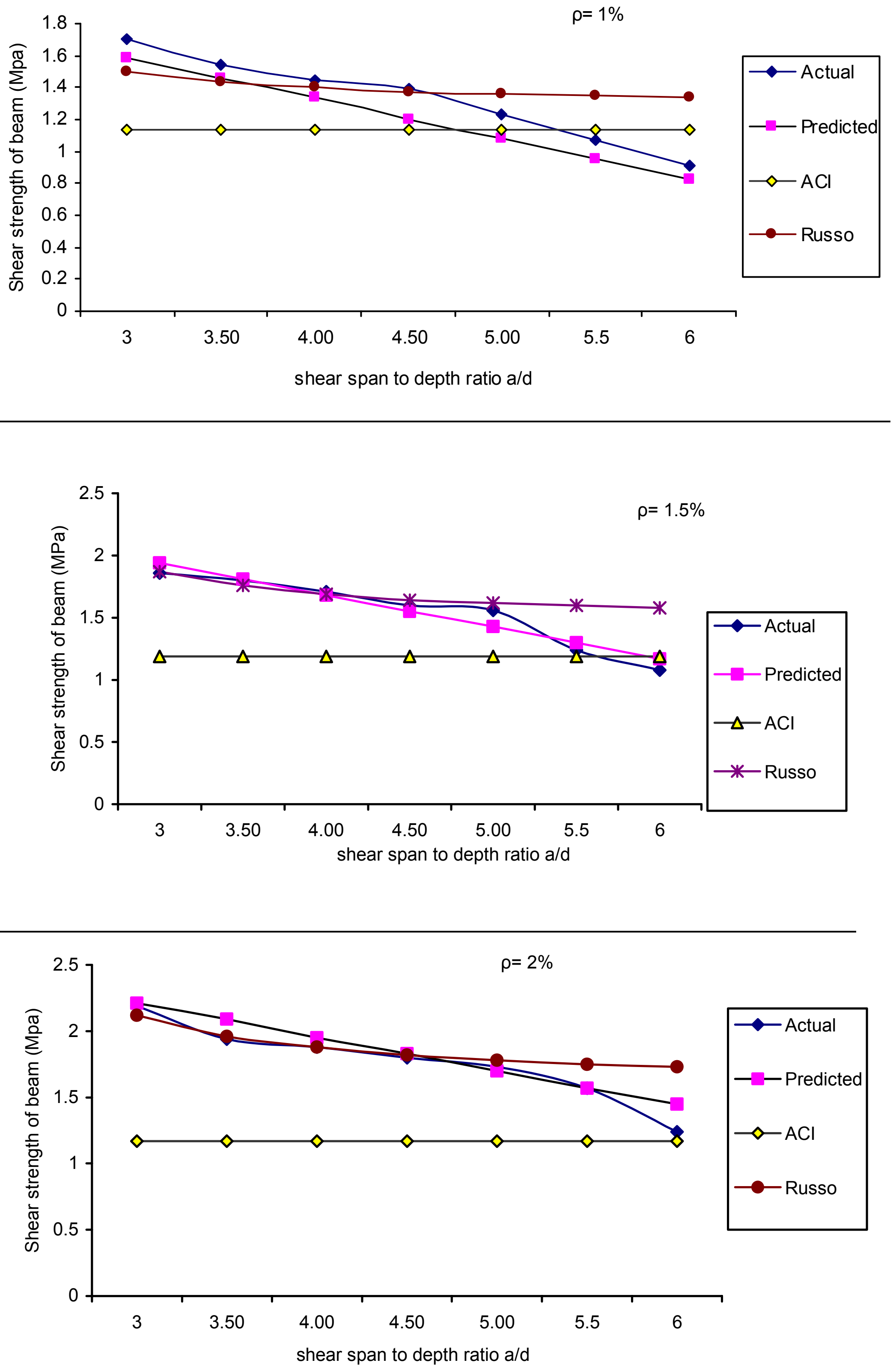


Figure 11. Comparison of actual shear stress of beams having shear reinforcement with the proposed equation and other models for $\rho \geq 1\%$

7. CONCLUSION AND RECOMMENDATIONS

From comparison of the actual and predicted values of shear stress of beams, the following results have been observed:

i. For beams without shear reinforcement, the proposed equation is conservatives for ρ = 0.75% and 1% and un-conservative for ρ =0.33%. For ρ=2%, the equation gives closer values to the actual observations.

ii. For beams with shear reinforcement, the proposed regression equation is conservative for ρ = 0.33%, 0.75% and un-conservative for ρ =1%.

iii. For ρ=1.5% and 2%, the equation gives closer values to the actual.

From comparison of the predicted values of shear stress and values proposed by ACI, the Russo *et al.* equation, and the Bazant *et al.* equation, the following results have been observed:

i. The Bazant *et al.* equation is un-conservative in estimating the shear stress for the HSRC beams without web reinforcement as it overestimates the shear stress for all values of longitudinal steel.

ii. The Russo *et al.* equation is more conservative as it underestimates the shear stress of the HSRC beams without web reinforcement.

iii. The ACI-318 equation for shear stress of HSRC beams gives some reasonable values when compared with the actual and predicted values.

iv. The Russo *et al.* equation, on the other hand, is un-conservative for shear stress of HSRC beams with web reinforcement.

v. The proposed regression equation better estimates the shear stress of beams as compared to the other models of ACI, Russo and Bazant. However, this may be due to the fact that the equations are based on the data of the tests' results. Hence, for generalization of the regression equation to other sets of tests' data, more experimental research is required.

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