# EVALUATION OF SHEAR STRENGTH OF HIGH STRENGTH CONCRETE CORBELS USING STRUT AND TIE MODEL (STM)

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## الخلاصـة:

تم – في هذا البحث – استخدام طريقة نمذجة الضغط والشد (STM) كطريقة تحليل جديدة من أجل تصميم أجزاء المباني الخرسانية التي لا تنطبق عليها متطلبات تطبيقات نظرية الانحناء المعروفة . وحيث إن الطنف (الحواف الناتئة) الخرسانية المزدوجة تعتبر أحد نماذج العناصر الخرسانية غير الأنثنائية التي تكون فيها نسبة طول ذراع القص إلى عمق المقطع أقل من1 ، فإن هذا البحث يقدم نتائج دراسة تحليلية لتسعٍ من الطنف الخرسانية المزدوجة ذات المقاومة العالية باستخدام طريقة STM التي غلب على استخداماتها فيما مضى استخدامها فقط لدراسة الطنف الخرسانية المزدوجة ذات المقاومة الخرسانية الاعتيادية . ولمقارنة النتائج النظرية ) باستخدام طريقة STM ( مع النتائج الاختبارية تم إجراء الاختبارات المعملية على هذا العدد من الطنف الخرسانية المزدوجة ، وكانت النتائج المعملية لتقدير قيم مقاومة القص في هذه العناصر الخرسانية (الطنف المزدوجة) مقاربة جداً للنتائج النظرية التي تم الحصول عليها من طريقة نمذجة الضغط والشد، لذا STM فإن هذا البحث يعتبر محاولة إضافية لتأكيد صلاحية استخدام طريقة STM كطريقة بديلة لتصميم الطنف الخرسانية المزدوجة ذات المقاومة العالية.

## ABSTRACT

The Strut and Tie Model (STM) has been used as an emerging analysis tool for the design of disturbed regions and non- flexural members in concrete structures, where ordinary flexural theory cannot be adopted. Double-corbels are typical non-flexural members and the shear arm length to depth ratio at the critical section is often less than 1. STM has been mostly used for the design of normal strength concrete double-corbel. In this research nine double-corbels of high strength concrete have been analyzed on the basis of STM, for assumed external loads and geometry. These corbels were later tested in the laboratory to compare the theoretical and actual failure loads. The experimental values have been observed to be quite close to the theoretical values of shear strength of the corbels on the basis of STM. The research therefore attempts to verify STM as an alterative design method for the high strength concrete double-corbels.

***Key words***: high strength concrete, disturbed region, strut and tie model, corbel

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## 1. INTRODUCTION

The analysis and design of non-flexural members, like dapped ended beams, corbels, deep beams, pile caps and openings in slabs, becomes difficult with ordinary flexural analysis as the true behavior of the member cannot be anticipated with the ordinary flexure theory. Different empirical approaches have been used for the design of such non-flexural members.

The Strut and Tie Model (STM) is one of the rational and relatively simple design approaches for non-flexural members, where traditional beam theory cannot be applied [1]. The STM is based on lower bound theory of plasticity, which assumes that steel and concrete are frequently plastic at the limit state. Efficiency factors are then applied to uni-axial strength of concrete to account for concrete softening and cracking [2] .

On the basis of its rationality and simplicity, STM has been adopted by Canadian Code in 1984, and by AASHTO LRFD bridge design in 1994 [3]. It was incorporated in the ACI building Code 318-02 as Appendix A, and later STM was recommended as an optional design procedure for disturbed region in ACI 318-05 Building Code [4].

Brackets and corbels are short-haunched cantilevers that project from the inner face of columns or concrete walls to support heavy concentrated loads of pre-cast beams, gantry girders, and other pre-cast system loads. The ratio of shear arm to depth is often less than 1.0. The nonlinear stress behavior of the short member is thus affected by the shear deformation in the elastic range and consequently the shear strength of the section becomes an important parameter for design consideration [5].

The ACI current design procedures for corbels are based on shear friction and empirical relationship based on the flexural capacity of the section. The two basic equations allowed for design of the corbels by ACI-318 are given as:

$$V_u = \phi \mu A_{vf} f_y \tag{1}$$

$$V_u = \frac{M_u}{a} \tag{2}$$

where

$$M_u = \phi \mu A_{st} f_y \left(d - \frac{A_{st} f_y}{1.7 f'_c b}\right) \tag{3}$$

The lesser of the two values is used for design.

The ACI code further restricts the shear resistance of the section as $V_u \leq \phi f b d$

where $f = 0.2 f'_c$ but not more than 5.5 N/m$^2$ (800 psi).

Saffi *et al* [6] and Foster *et al.* [7] used an efficiency factor for the plastic truss model developed by Rogowsky and Macgregor [8] for a possible failure model. The failure of corbels can take place due to crushing of concrete in the compressive struts, failure due to yielding of steel, splitting failure, and anchorage failure.

The concrete efficiency factor proposed by Saafi *et al.* [6] was based on the concrete strength, and shear span to depth ratio. They observed that the experimental values are well predicted by the revised model.

Shawi *et al.* [9] gave a simplified approach for the design of HSC corbels. They have observed that the ACI code underestimates the load carrying capacity of the HSC corbels. They proposed a new equation for the load carrying capacity of HSC corbels with shear span to depth ratio ≤ 1.0.

The Strut and Tie Model has been used for the design of disturbed regions in RC members, including corbels, very selectively, since the concept was initially introduced by Schlaich, *et al.* [2] and no generalized design procedure based on STM was developed for many years. However, the ASCE-ACI Committee-445 report on Shear and Torsion [14] provided a detailed procedure based on STM, for the design of disturbed region. The STM has been widely used for the design of deep beams [10-13], quite successfully.

The Strut and Tie procedure is relatively straightforward and involves the following three key steps [14]:

1. Develop a Strut and Tie Model (STM). The struts and ties serve to condense or replace the real stress field by resultant straight lines and concentrate their curvature in nodes.
2. Calculate the strut and tie forces from equilibrium.
3. Dimension the struts and ties for internal forces, with due consideration of crack width and orientation.

The design of high strength concrete corbels using the Strut and Tie Model and its comparison with actual laboratory tests is not readily available in the literature as most of the available data is based on normal strength concrete with compressive strength of 40 MPa or less.

In this research, nine two-way corbels, with three values of compressive strength of concrete (45, 52, and 59 MPa) were cast. The minimum area of steel allowed by the ACI was used. The theoretical shear capacity of the corbels was worked out on the basis of STM. The shear strength (load carrying capacity) of these samples was tested in the laboratory under monotonic loading. The results were compared with the theoretical capacity of specimen based on STM. The results have revealed that STM gives fairly realistic values of the shear capacity of the two-way corbels.

## 2. RESEARCH SIGNIFICANCE

Truss models are widely used for the design of non-flexural members in concrete structures. The research has focused on the application of Strut and Tie Model (STM), for the design of high strength concrete double-corbels. The research objectives can be summarized as follows;

1. To develop the Strut and Tie Model (STM) for double-corbels on the basis of ACI-318 procedure and design guideline.
2. To determine the actual load carrying capacity of the double- corbels under external monotonic loads for the given concrete compressive strength, corbel geometry and minimum steel ratio recommended by ACI.
3. Comparison of the actual laboratory test values and theoretical load carrying capacity of the corbels based on STM.

The research attempts to verify STM as an alternative design method for non-flexural disturbed regions in concrete structures. The comparison will provide more freedom to structural designers in applying their professional judgment for developing the basic truss model for the design of disturbed regions in concrete structures.

## 3. PROBLEM

A high strength concrete double-corbel projecting from 9 in × 9in ( 23 cm × 23 cm ) square column was provided with minimum steel reinforcement required by the ACI as $A_{s\,\min} = 0.04\frac{f_c'}{f_y}.b.d$ , for three values of $f_c'$ .i.e. 6500psi, 7500 psi, and 8500 psi (45,52 and 59 MPa). The reinforcing steel used was deformed bars of 60 000 psi (414 MPa) yield strength. Loads were applied at each end at 4.5 in (11.43 cm) from the face of column and corbel. The dimensions of the corbel are shown in Figure 1. The loading arrangement is shown in Figure 2. The corbels were then tested under monotonic load in the Structural Laboratory of Engineering University located at Taxila-Pakistan. The details of corbel forms and steel reinforcement are shown in Figure 3. To record the strain of the ties, embedment gauges were placed along the steel bars as shown in Figure 4.

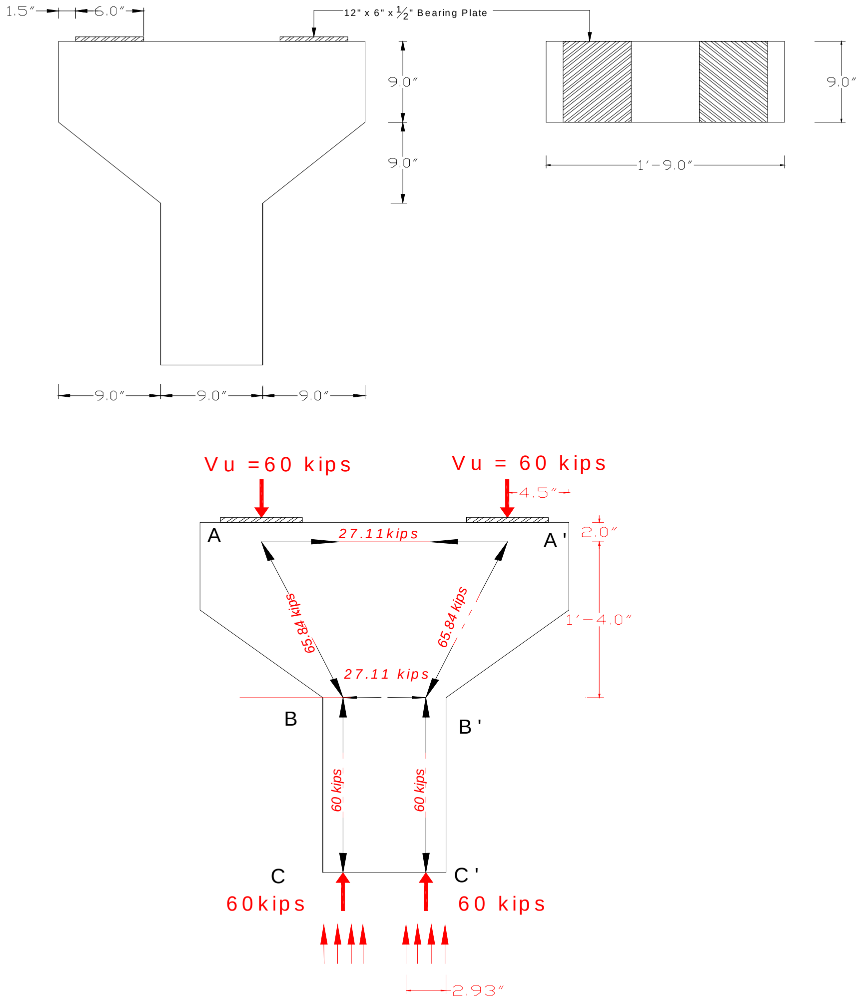

*Figure 1. Geometry of the truss and members for ces*

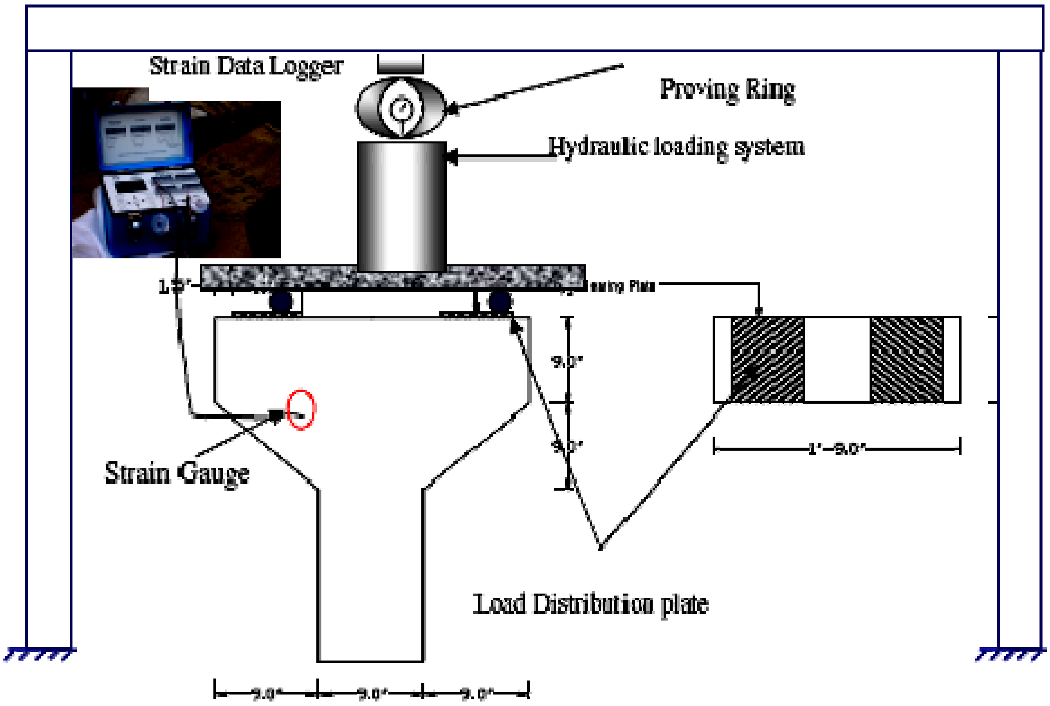

*Figure 2. Loading arrangement for two way corbel*

*Figure 3. Corbel form work and reinforcement*

*Figure 4. Concrete embedment gauges placed along the steel bars to measure the strain in ties*

## 4. ANALYTICAL SOLUTION OF THE TWO CORBELS USING STRUT AND TIE MODEL

To determine the shear capacity of the corbels for the provided steel and compressive strength of concrete, the corbel was assumed to take a total load of 120kips (534KN), equally transferred to both ends of the corbels. The following steps as recommended by the ACI-ASCE Committee are adopted for the design against an assumed load of 60 kips (267 kN) on each end to check the suitability of the minimum steel provided. The design example has been solved for $f_c'$ = 6500 psi (45 MPa)

### Step No. 1: Determine the Bearing Plate Dimension

Bearing plate measuring 9in × 6in (23 cm × 15.24cm) is selected to transfer the load evenly to the corbel projection, the bearing area of plate is 54 in$^2$ (348 cm$^2$). The corbel is designed to carry a load of 60kips at each end.

The bearing stress = 60×1000/54 = 1111 psi (7.66 MPa)

Thus, the allowed limits of the bearing stress $= \phi(0.85\beta_n f_c')$ = 0.75 × 0.85× 0.80 × 6.5 = 3.315 Ksi (22.86 MPa), which is more than the applied stress.

### Step No. 2: Choosing the Corbel Dimension

The depth of column face is 9 in (23 cm). ACI requires that the depth outside the bearing must be at least half of the depth of column. In our case the load is applied at 4.5 in (11.5 cm) from the face of corbel, which is half the column dimension. Hence, the minimum requirements are fulfilled.

### Step No. 3: Establish Strut and Tie Model

The geometry of assumed truss is shown in Figure 1. The centre of the tie is assumed to be 2 in below the top of the corbel. Hence, $d = 18 - 2 = 16$ in (40cm). The horizontal strut *BB′* is assumed to lie in the horizontal line at the corbel column joint.

The location of strut *CB* centerline can be found by calculating the required strut width ($a$) in terms of the compressive force in strut *CB*, $N_{CB}$, and the strut stress limit.

The strut *CB* force is $N_{CB}$ = 60 kips

The limit stress on the nodal zone *B* (also strut *CB*) is $\phi f_{cu} = \phi(0.85\beta_n f_c')$

The width of the truss is given as $a = \dfrac{N_{CB}}{\phi . f_{cu} b}$

This fixes the geometry of the truss.

**Step No. 4: Truss Forces**

The required forces in all the members of the truss are given as follows;

| Member | ***AA′*** | ***AB*** | ***BB′*** | ***BC*** |
|---|---|---|---|---|
| **Force (kips)** | 27.11 | −65.84 | −27.11 | −60 |

Note that the positive sign indicates tension and the negative sign indicates compression.

**Step No. 5: Design of Tie**

The area of reinforcement required for tie *AA′* is $A_{s\,required} = \dfrac{N_{AA'}}{\phi . f_y}$

$$A_{s\,\min} = 0.04 \frac{f_c'}{f_y} .b.d$$

**Step No. 6: Check the Struts**

The struts are checked by computing their widths and checking whether they will fit in the space available.

The stress of the diagonal strut *AB* is limited to $\phi f_{cu} = 0.75(0.85\beta_s f_c')$

Hence, the required width for strut *AB* is $\dfrac{N_{AB}}{\phi . f_{cu} b}$

The stress of the vertical strut *CB* and horizontal strut *BB′* is limited to $\phi . f_{cu} = 0.75(0.85\beta_s f_c')$

Hence, the required widths for strut *BB′* is $\dfrac{N_{BB}}{\phi . f_{cu} b}$

Thus, it can be checked whether the truss lies within the corbel or not.

**Step No. 7: Design of Nodal Zone and Check for Anchorages**

The width (*a*) of nodal zone *A* was chosen to satisfy the stress limits on the nodal zone.

To anchor tie *AA′*, the horizontal loop is used. To satisfy the nodal zone stress limit, the tie reinforcement must engage an effective depth of concrete at least equal to

$$\frac{N_{AA}}{\phi \cdot f_{cu} b} = \frac{N_{AA'}}{\phi (0.85\beta_n f_c')b} \ (N)$$

The required anchorage length for tie *AÁ* is $l_{dh} = \lambda \dfrac{1200 d_b}{\sqrt{f_c'}}$

## 5. LABORATORY TESTING AND OBSERVATIONS

The corbels were tested under two point loads applied at the centre line of bearing plates at both ends. The loads were applied through hydraulic system, transferring the load through calibrated proving ring as shown in Figure 2. The loads were gradually increased and the cracks developed in the corbels were closely observed. The cracks started from the outer edge of the plates and gradually extended down towards the connection of corbel and column, showing typical shear cracks. With further increase of applied load, the crack surface widened and became more prominent, ultimately causing failure of the corbel. The corbels failed mainly due to failure of compression struts as shown in Figure 5 and the inclination of the struts are falling in the range of $67^\circ$ to $72^\circ$, in comparison with the theoretical value of $67^\circ$. Failure was more brittle and sudden due to high strength of concrete. The theoretical shear capacity of the corbels for minimum shear reinforcement, material stresses and truss geometry was determined by trial and error in a MS Excel spread sheet and compared with the actual values in Table 1. The theoretical and observed values of the shear capacity are relatively more closer for $f_c'$ = 6500 psi (45MPa), but the variation is more in case of high strength concrete $fc'$ = 7500 and 8500 psi (52, 59 MPa). The strut angle in case of high strength concrete corbels is also steeper. This may be mainly due to reduction in the aggregates interlocking at higher strength of concrete. Concrete has cracked across the aggregates rather than at the contact points of the aggregates and mortar.

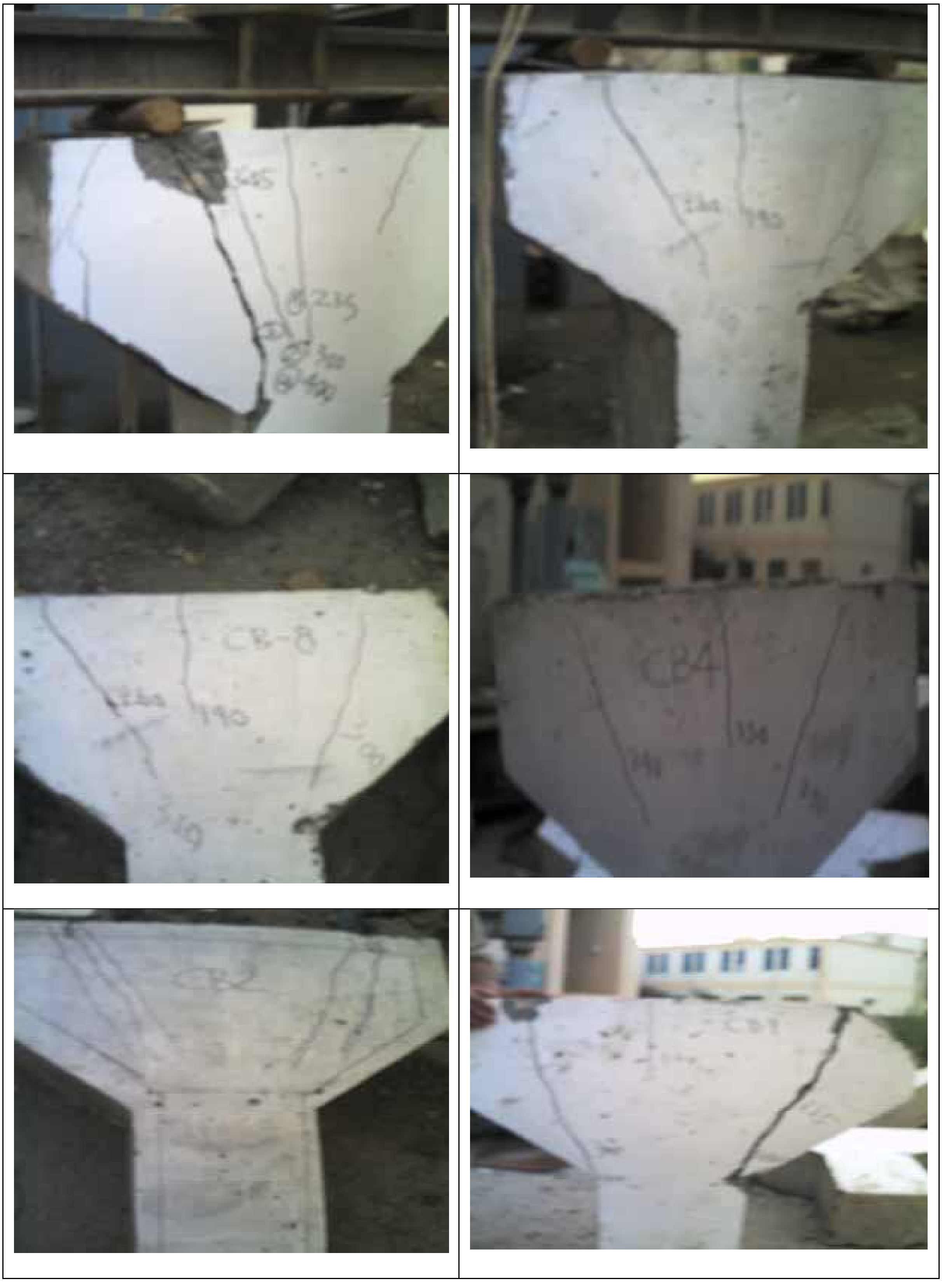

*Figure 5. Typical shear failure of the two way corbels*

**Table1. Details of Truss Forces, Theoretical Failure Loads on STM Basis and Actual Failure Loads of High Strength Corbels**

| **Corbel** | **CB-1 to CB-3** | **CB-4 to CB-6** | **CB-7 to CB-9** |
|---|---|---|---|
| **Technical Parameters** | | | |
| *fc′*<br>(ksi) | 8.5<br>(59MPa) | 7.50<br>( 52Mpa) | 6.5<br>( 45 MPa) |
| As provided (in$^2$)<br>( SI units) | 3#4<br>( 0.60)<br>(3#10) | 3#4+1#3<br>( 0.71)<br>( 3#10+1#13) | 4#4<br>( 0.81)<br>( 4#13) |
| Theoretical Shear capacity<br>(kips) | 60<br>( 269 kN) | 70<br>( 311kN) | 80<br>( 356kN) |
| **Truss Forces & Geometry** | | | |
| Strut (kips) | 65.84<br>( 292kN) | 76.84<br>( 342kN) | 87.84<br>( 390kN) |
| Tie (kips) | 27.11<br>( 120kN) | 31.70<br>( 141kN) | 36.28<br>( 161kN) |
| Strut AA′ width (in) | 2.05<br>( 5.2cm) | 2.32<br>(5.89cm) | 2.68<br>( 6.8cm) |
| Strut BB′ width (in) | 1.80<br>( 4.57cm) | 1.79<br>( 4.5cm) | 1.77<br>( 4.49cm) |
| **Strut Angle** | | | |
| Theoretical | 67.55 | 67.58 | 67.62 |
| Actual | 67 | 66.97 | 72.75 |
| **Failure Shear Loads** | | | |
| Theoretical (kN) | 60<br>( 269 kN) | 70<br>( 311kN) | 80<br>( 356kN) |
| Actual | 63<br>( 280kN) | 68<br>( 302kN) | 72<br>( 320kN) |
| % variation | 5% | 3% | 10% |
| Strain in tie ( $10^{-6}$ m or $\mu_s$) | 209 | 285 | 300 |

## 6. CONCLUSIONS

The comparison of the theoretical and observed failure values of the corbels shows very slight variation, which provides reasonable arguments for the suitability of STM for the design of two way corbels. However generalization of the method for design of two-way corbels would certainly need more research and empirical evidence. Further research on the application STM for the design of high strength concrete two-way corbel is therefore recommended.

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