EVALUATION OF THE SHEAR STRENGTH OF FOUR PILE CAP USING STRUT AND TIE MODEL (STM)

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ABSTRACT

Strut and Tie Model (STM) has been widely used for the design of disturbed region and non flexural members in RC structures. Pile cap is typically a disturbed region with small length to depth ratio, hence ordinary flexural theory for beams cannot be applied to it. In this research, six pile caps were designed for certain theoretical ultimate loads on the basis of STM. These pile caps were tested on four simply supporting piles. Loads were applied at the centre of pile cap. The experimental values were compared with the theoretical capacities of the pile caps on the basis of STM. It has been observed that STM has provided a reliable solution for predicting the shear strength of the four-pile caps and the experimental values fall very close to the theoretical values based on STM.

Key Words: disturbed region, Strut and tie model, pile cap, STM

I. INTRODUCTION

Structural elements like deep beams, pile caps, corbels, ribs and end zones of pre-stressed girders cannot be treated as flexural members for design purpose. Due to their specific geometry, small length to depth ratios, the traditional Bernoulli's theorem applicable for beam region (B-region) cannot be used for design of these members (Wight *et al*., 2003). The non linear members develop internal arching which enhances shear strength considerably. The strut induced by the arch action transmits the load to the supports. Hence these non linear members can be modeled as analogous struts.

STM evolved in the early 1980s in Europe (Schlaich *et al*., 1987, Marti., 1985, 1987). Strut and Tie Model (STM) is a relatively simple design approach for non flexural members, which is based on the equilibrium of internal forces in a structure, when it is subjected to external loads. The compatibility requirement of the strains is neglected, which makes it relatively simpler as compared to other design approaches.

STM has been adopted by the Canadian Code in 1984 and AASHTO LRFD bridge design in 1994 (FIP Congress., 1994). The use of STM has been permitted by ACI Code 318-02 and later it was incorporated as an alternative design method in ACI-318-05 (ACI building code. 2005) for disturbed regions in reinforced concrete structures.

The STM is based on lower bond theory of plasticity assuming that steel and concrete are frequently plastic and efficiency factors are applied to uniaxial strength of concrete to account for concrete softening (Clarke., 1987). The STM design is not unique as it depends on the shape of non flexural structure, material, design perception and understanding of the structure. However the method has opened a new venue for research in the design of disturbed regions.

The Strut and Tie procedure is relatively straightforward and involves three key steps;

1. Develop Strut and Tie Model (STM). The strut and ties serve to condense or replace the real stress field by resultant straight lines and concentrate their curvature in nodes.
2. Calculate the strut and tie forces to satisfy equilibrium.
3. Determine dimensions of the strut and ties for internal forces.

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4. Determine the reinforcement to resist the ties and check the capacity of the struts against the internal forces.
5. Check the capacity of the nodal region for anchorage of the steel bars.

Pile caps Pile caps are important structural elements with the function to transfer basically the stresses of the columns to a group of piles.

Historically pile cap has been designed by the following four methods;

1. The Truss analogy.
2. ACI Code
3. AASHTO LRFD Standard and
4. Strut and Tie model.

The literature on the design of pile caps is however limited. Clarke *et al* (1987) presented the results of 15 tests performed on four pile caps carried out at the Cement and Concrete Association in the United Kingdom. He used the provisions of British building code CP110 at that time in 1973 and reported that the code cannot address the design problem of four pile caps. However the experimental results of Clarke still have research significance to understand the problem.

Gogate and Sabnis (1984) suggested that the provisions of ACI 318-77 are inadequate for the design of pile caps.

Adebar, Kuchma and Collins (1990), carried this work further by performing an experimental study of the strut-and-tie model in pile cap design and concluded that STM more accurately predict the behavior of deep pile caps.

Adebar and Zhou (1993), proposed an empirical relationship defining the maximum allowable bearing stress for design. They recommended that STM is the best design method for pile caps.

Suzuki (1999) studied the influence of edge distance, bar arrangement, and taper on pile cap performance.

The joint committee report of ACI-ASCE 445 (1998), on shear and torsion has given a detailed commentary on the use of STM for design of disturbed region, including pile caps.

The research of Mariam (2008), three pile caps showed that the struts methods give conservative results and an overall margin of safety of 12% was observed.

According to the Canadian building Code the strut-and-tie model is used for determining the reinforcement area and anchorage requirements of pile caps. The effective footing depth is governed by the one- and two-way shear requirements of CSA A23.3-94 (Cls. 11.3 and 13.4), and pile and column areas are governed by the bearing stress requirements of CSA A23.3-94-Cl. 10.8 (Miriam *et al*., 2004).

The use of STM for any disturbed region like a pile cap depends on the selection of an appropriate truss model as there is no unique STM for any structure. Hence an attempt has been made in this research to assume an STM for the given load and then test its shear strength to check the accuracy of the assumed model.

The limitation of the study comes from the inherent weakness of the STM as an iterative process as there is no unique STM for any particular disturbed region in concrete. However the recent contemporary research in the disturbed region and truss models is providing better knowledge to structural designers to use their intuition and rationality for developing most the appropriate truss models. This work is an attempt in this direction.

In this research six samples of four-pile caps of size 75 cm x 75 cm x 23 cm were designed with STM for assumed external loads. Two mix designs of concrete were used with three samples from each mix. These four-pile caps were then tested with a concentrated load at the centre and the actual failure loads were observed. These experimental loads were compared with the theoretical shear capacity of the pile caps designed by STM. The results have shown that the assumed STM has reasonably predicted the shear strength of the tested pile caps; however more experimental work is required to generalize the STM.

II. RESEARCH SIGINIFICANCE

The research objectives are summarized as;

1. Understanding and analyzing the shear behavior of four pile caps as disturbed region.
2. Design of four pile caps with the help of Strut and Tie Model for certain assumed external load.
3. Comparison of theoretical and actual shear failure loads of the pile caps and check the safety provided by STM.
4. The results will provide evidence for using the STM for shear design of four pile caps with more accuracy and confidence by the designers.

III. TEST PROGRAM AND ANALYSIS

A four pile cap of size 2.5 ft (75 cm) × 2.5 ft (75 cm) and 9 in (23 cm) deep supported on four piles of 6 in (15cm) diameter at the corners was required to be designed to carry a theoretical load of 100 kips (22.5 KN) and 120 (27 KN) for the concrete compressive strengths $f'c$ of 3000 psi (20.68 MPa), 4000 psi (27.6 MPa). The theoretical model of four-pile cap is shown in Fig. 1.

The three dimensional analysis of the pile cap may provide better results, however due to symmetry

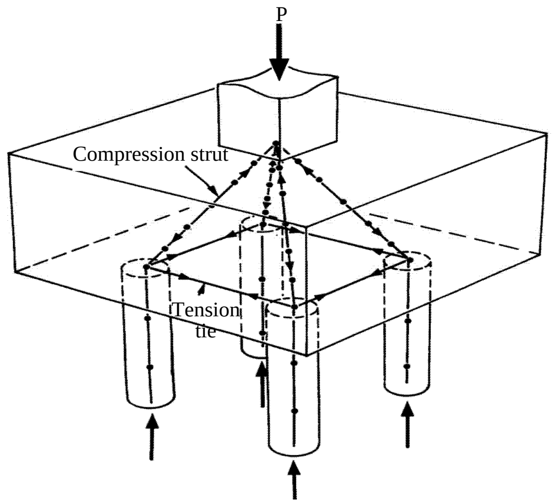

Fig. 1 Theoretical 3 D- Strut and Tie Model of Four-pile cap

of the pile cap in both direction, and the main flow of the forces parallel to the horizontal axis, a planar 2-dimensional truss model has been assumed. The transfer of forces is assumed to flow in the horizontal short direction parallel to the plan of the pile cap. The diagonal distance between the pile and column, is relatively more that the distance between the pile caps, hence the forces transferred in the third dimension are neglected for convenience of the designer. The main reinforcement has been provided in the horizontal direction hence planar STM has been assumed.

Figure 2 shows the initial 2 dimensional, strut-and-tie model (or truss) assumed for analysis and design of this pile cap. The broken lines represent compression members (struts) and the solid lines represent tension members (ties). For simplicity, the nodes (intersection of the struts and ties) are shown as dimensionless points.

A trial value must be selected for the depth of the truss *dv* to solve for the truss member forces. With these forces, the dimensions of the struts, ties, and nodal zones can be established, and the value for *dv* can be verified or Modified with a second iteration.

Because of the small span-to-depth ratio (approximately 1.33) for the left portion of the cap, only a single strut is used between the concentrated load and the support. The right portion of the cap also requires a single strut between the concentrated load and the support. To control the use of shallow angle struts, ACI Code requires a minimum angle of 25 degrees between struts and ties. In this case our angle is 29.05 degree.

1. SOLUTION FOR THE LEFT PORTION OF THE CAP

It is convenient for the analysis of Node 2 to

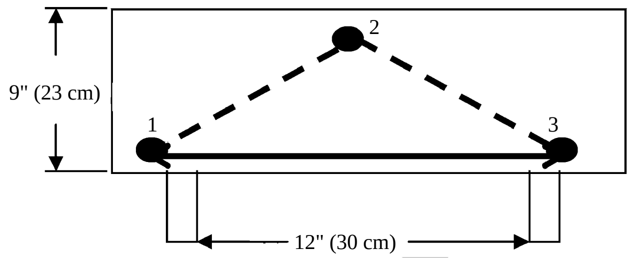

Fig. 2 Assumed Strut and Tie Model (STM)

break the concentrated load at the top of the cap into two parts and solve the left portion of the cap.

Step 1. Establish The Truss Geometry and Truss Member Forces

The depth of truss is given as dv = 8 in. – 2(1.5 in.) = 5 in. (12.8 cm)

tan α = (5 in. / 9 in.) implies that α = 29.05 degrees

From equilibrium at Node 1

$$\Sigma(FY) = 25\text{ k} - F12\ (\sin\alpha) = 0 \quad (1)$$

$$F_{12} = -51.50\text{ Kips (11.58 KN)} \quad (2)$$

$$\Sigma(Fx) = \text{F14} - \text{F12}\ (\cos\alpha) = 0 \quad (3)$$

$$F_{14} = 45\text{ kips (10.12KN} \quad (4)$$

Figure 3 gives the geometry of Node 1. Since Fig. 3 shows the procedure to calculate the width of Strut1-2, effective compressive strength for a node is defined as:

$$fcu = (0.85)\text{ bn } f'c \quad (5)$$

Node 1 is a compression-compression-tension (CCT) node, so βn = 0.8. Thus, the effective compressive strength for Node 1 at nominal conditions is

$$fcu = (0.85)\ \beta n\ f'c = (0.85)(0.80)(3.0\text{ ksi})$$
$$= 2.04\text{ ksi (14.07 Mpa)} \quad (6)$$

Use this nominal strength and φ = 0.75 to check stress at the base of the node

$$f_{base} = Rb/(b_w)(lb_1) \quad (7)$$

$$f_{base} = 25/6 \times 6 \quad (8)$$

$$f_{base} = 0.69\text{ ksi} < 2.04\text{ ksi}$$
$$(4.75\text{ MPa} < 14.07\text{ MPa})\text{ OK} \quad (9)$$

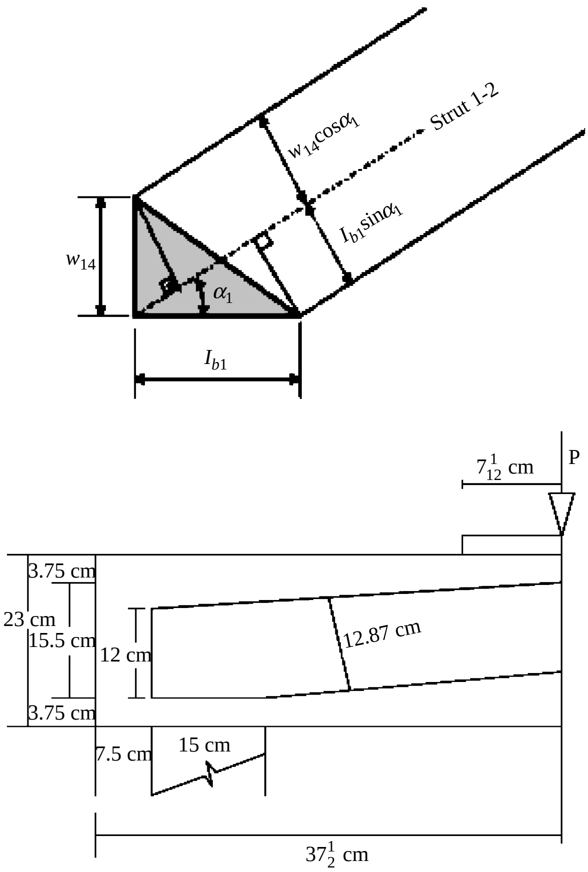

Fig. 3 Proposed truss model for left half of the pile cap

Also, find the width of Tie 1-4, which defines the height of Node 1

$$W_{14} = F_{13}/(\phi \times b_w \times f_{cu} \times 1) \quad (10)$$

$$W_{14} = 45/(0.75 \times 6 \times 2.04) = 4.9 \text{ in (12.5 cm)} \quad (11)$$

$$W_{14} = 4.90 \text{ in} = 5\text{in (approx) (12.5 cm)} \quad (12)$$

The effective compressive strength *fcu*, for Strut in Fig. 2 , probably controls the stress on the inclined face of Node 1. This value is determined using the same expression as given previously for a node, with βs substituted for βn. For Strut, use $\beta s = 0.75$, which assumes that a minimum amount of reinforcement will be provided across the strut as required in Section A. 3.3 of the Code

$$fcu = 0.85\ \beta s f'c = (0.85)(0.75)(3.0 \text{ ksi})$$
$$= 1.91 \text{ ksi (8.21 MPa)}$$

Use the geometry of Node 1 shown in Fig. 3 to determine the width of Strut.

$$Ws(1\text{-}2) = W_{14}(\cos\alpha) + lb_1 (\sin\alpha)$$
$$= (5 \text{ in})(0.87) + (6 \text{ in.})(0.48)$$
$$= 7.28 \text{ in. (18.5 cm)} \quad (13)$$

Checking the strut capacity

$$\phi Fns(1\text{-}2) = \varphi fcu \times ws_{(1\text{-}2)} \times bw$$
$$= (0.75)(1.91 \text{ ksi})(7.28 \text{ in.})(6 \text{ in.})$$
$$= 63 \text{ k} > 52 \text{ k (280 KN} > 231 \text{ KN) OK} \quad (14)$$

Step 2. Check Maximum Shear Permitted by the ACI Code

ACI-Code Section 11.8.3 defines an upper limit for the shear force permitted in a deep beam. With the centroid of Tie 1-4 established, the effective flexural depth of the Cap d is

$$h - (w_{14}/2) = 5.5 \text{ in.} \quad (15)$$

Thus, the check of Code Section 11.8.3 requires

$$Vu \le \varphi Vn(\max)$$
$$= \varphi(10)\sqrt{f'c}\,b_w d$$
$$= (0.75)(10)\sqrt{3000} \text{ psi (30 in.)(5.5 in.)} \quad (16)$$

$$Vu = 25 \text{ k} \le \varphi Vn(\max)$$
$$= 68 \text{ Kips (15.28 KN) OK} \quad (17)$$

Step 3. Check Maximum at Node 2 as Similar to Node 1

Check the stress on the top face of Node 2 (CCC node) using $\beta n = 1.0$

$$fcu_{(2)} = 0.85\ \beta n f'c = 2.55 \text{ ksi} \quad (18)$$

$$f_{Top} = 50/6 \times 6$$
$$= 1.38 \text{ ksi} < 2.55 \text{ ksi (9.5 MPa} < 17.6 \text{ MP) OK} \quad (19)$$

$$f_{ver.face} = 45/3 \times 6$$
$$= 2.5 \text{ ksi} < 2.55 \text{ ksi}$$
$$(17.24 \text{ MPa} < 17.6 \text{ MPa) OK} \quad (20)$$

Check capacity of Strut 1-2 at Node 2 (Critical end)

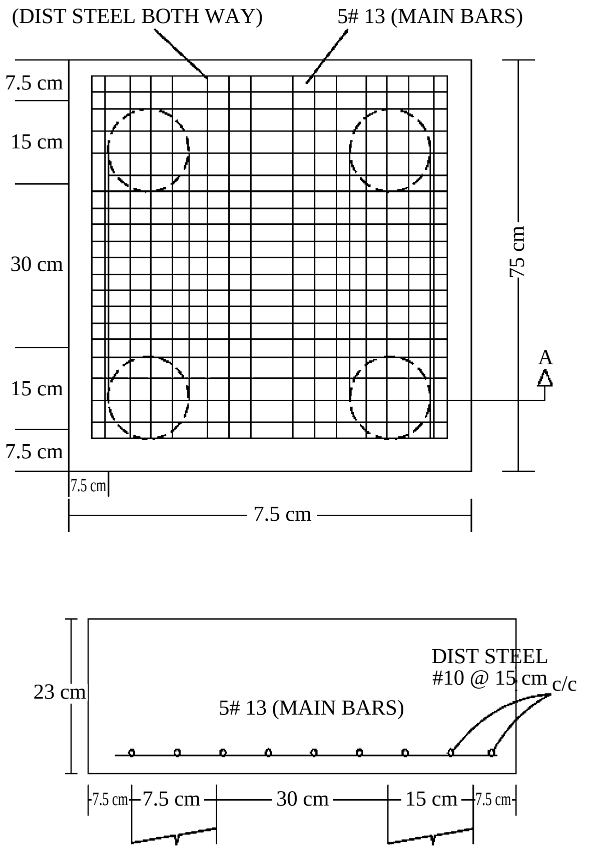

Fig. 4 Reinforcement details of the pile cap

$$\varphi Fns_{(1\text{-}2)} = \varphi fcu_{(1\text{-}2)} ws_{(1\text{-}2)}(b_w)$$
$$= (0.75)(2.55 \text{ ksi})(5.5 \text{ in.})(6 \text{ in.})$$
$$= 63 \text{ k} > 52 \text{ k } (280 \text{ KN} > 231\text{KN}) \text{ OK} \quad (21)$$

Step 4. Select Reinforcement for Tie 1- 3

Determine required area of reinforcing steel:

$As_{(\text{req})} = F_{13}/\varphi fy = 45/.75 \times 60 = 1$ in-square

Select 5 No. 4 bars (5#13) [As = 1 in. square] arranged in one row (bunched along the tie)

Use distribution steel: # 3 @ 6 in c/c (#10@15 cm c/c)

The above iteration is revised for $f'c$ = 4 Ksi (27.58 MPa) and $As = 1.2$ in^2 and the details are shown in Fig. 4.

IV. OBSERVATION

When loads were applied at the midpoint of the pile cap and gradually increased, flexural cracks appeared from the point of contact of the pile and cap.

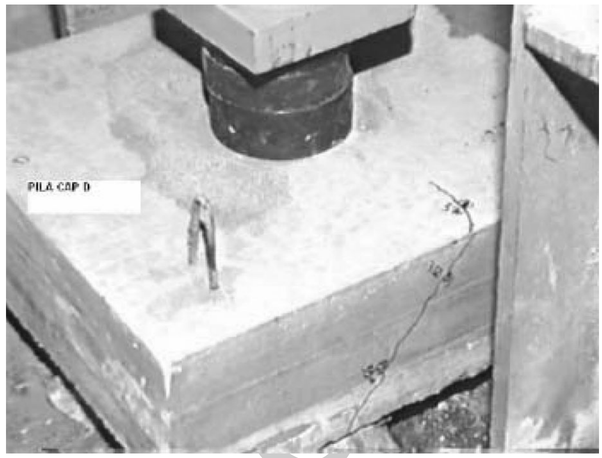

Fig. 5 Load applied at the centre of four-Pile cap. The Pile cap simply supported on four piles at the corner

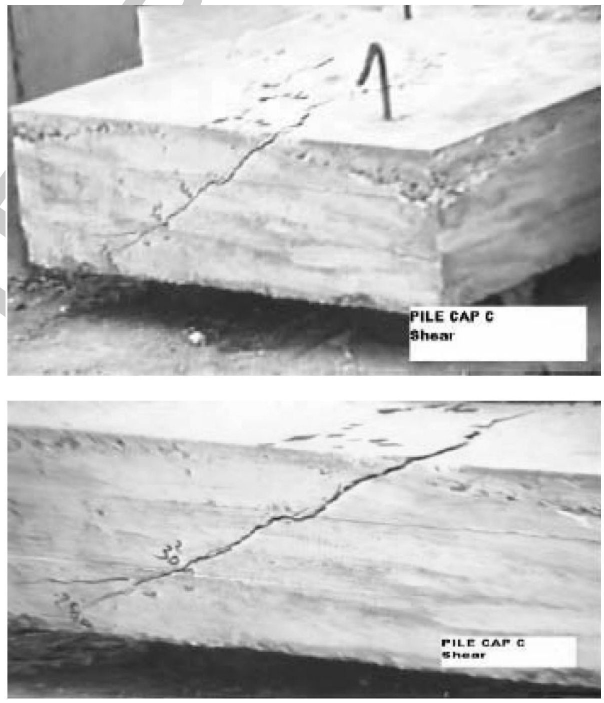

Fig. 6 Typical shear failure of pile cap

With further increase of loads, the cracks angles become shallower and the cracks extend towards the middle of the pile caps, which ultimately caused the failure of the pile caps as shown in Fig. 5.

The failure angles ranged from 32 degrees to 28 degrees and the average failure angle has been observed as 30 degrees.

The pile caps have mostly failed due to shear and the average failure angle has been observed as about 30° as against the theoretical value of 29.06°. The typical shear failure of the pile caps has been shown in Fig. 6.

Table 1 Details of Pile caps and comparison of experimental and theoretical Values (SI system)

Pile Cap & material stresses (Ksi)	Pile cap sizes and piledia (cm)	Truss geometry (cm)			Steel used	
	BxLxH	Depth d (cm)	Width b (cm)	Angle	Main	Dist
A fc' = 21 fy = 413	76 × 76 × 23 Dia. 15 cm	12.7	30	29.05	5#13	#10@15cm
B fc' = 21 fy = 413	76 × 76 × 23 Dia. 15 cm	12.7	30	29.05	5#13	#10@15cm
C fc' = 21 fy = 413	76 × 76 × 23 Dia. 15 cm	12.7	30	29.05	5#13	#10@15cm
E fc' = 27 fy = 413	76 × 76 × 23 Dia 15 cm	12.7	30	29.05	6#13	#10@15cm
F fc' = 27 fy = 413 G	76 × 76 × 23 Dia. 15 cm	12.7	30	29.05	6#13	#10@15cm
fc' = 27 fy = 413	76 × 76 × 23 Dia. 15 cm	12.7	30	29.05	6#13	#10@15cm

V. CONCLUSION

The theoretical and observed failure values of the pile

caps have been compared in Table 2.

The failure angles and loads of the piles caps fall closer to the theoretical angle of the assumed truss and their load bearing capacity of the pile caps designed on the basis of STM.

The average variation of 10% has been observed. Hence the assumed STM has provided a reasonable tool for predicating the shear capacity of four pile caps of the given geometry.

The selection of appropriate STM is critical to the theoretical results of STM and any change in the truss geometry would change the forces and results. Hence more experimental research and empirical evidence is required to generalize the STM for design. It is therefore recommended that more experimental research may be initiated for developing a relatively generalized and consensus based STM for four pile caps.

VI. LIST OF ABBREVIATIONS

dv = depth of assumed truss.

f_{cu} = Ultimate compression strength of node and struts.

For Struts $f_{cu} = 0.85\ \beta s\ f'c$

For Node $f_{cu} = 0.85\ \beta n\ f'c$

β_s = 1.00 for prismatic struts in untracked compression zones

β_s = 0.4 for struts in tension members

β_s = 0.75 struts may be bottle shaped and crack control reinforcement is included

β_s = 0.60 struts may be bottle shaped and crack control reinforcement is not included

β_s = 0.60 for all other cases

β_n = 1.00 when nodes are bounded by struts and/or bearing areas

β_n = 0.80 when nodes anchor only one tie

β_n = 0.60 when nodes anchor more than one tie

$f'c$ = Specified concrete compressive strength.

φ = Strength reduction factors = 0.75, for struts, ties, and nodes.

W_{14} = vertical depth of the truss

F_{13} = Compressive force in strut 1-3

W_{14} = Depth of strut 1-3

Vn = Nominal Shear strength

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Table 2 Comparison of theoretical and actual failure loads of pile caps

Pile Cap Title	Failure angle (degrees)		Load Carried by Two piles or one Truss (KN)		Vexp/VSTM
	Theoretical	Actual	Actual	Theo	
A	29.05	28	240	222	1.08
B	29.05	29	245	222	1.1
C	29.05	30	267	222	1.2
E	29.05	30	302	267	1.13
F	29.05	31	280	267	1.05
G	29.05	29	289	267	1.08
				Mean	1.10

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Manuscript Received: Dec. 21, 2007
Revision Received: Aug. 24, 2008
and Accepted: Sep. 24, 2008