Strength of Material

Shear Strain

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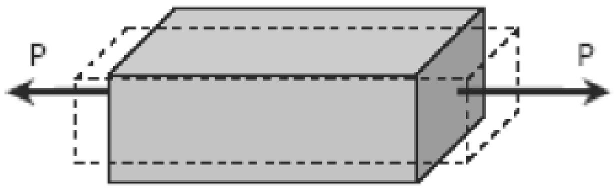
Shear Strain

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

Poisson's Ratio



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

BIAXIAL DEFORMATION

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \text{ or } \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1-\nu^2}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \text{ or } \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1-\nu^2}$$

TRIAXIAL DEFORMATION

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Relationship Between E, G, and v

$$G = \frac{E}{2(1+\nu)}$$

Bulk Modulus of Elasticity or Modulus of Volume Expansion, K

- The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as:

$$K = \frac{E}{3(1-2\nu)} = \frac{\sigma}{\Delta V / V}$$

- Where V is the volume and ΔV is change in volume. The ratio ΔV / V is called volumetric strain and can be expressed as

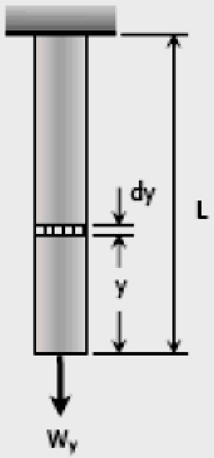
$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1-2\nu)}{E}$$

- **Problem 205** A uniform bar of length L, cross-sectional area A, and unit mass ρ is suspended vertically from one end. Show that its total elongation is $\delta = \rho g L^2 / 2E$. If the total mass of the bar is M, show also that $\delta = MgL/2AE$.

Solution 205

$$\delta = \frac{PL}{AE}$$

From the figure:
$\delta = d\delta$
$P = W_y = (\rho A y)g$
$L = dy$



$$d\delta = \frac{(\rho A y)g\, dy}{AE}$$

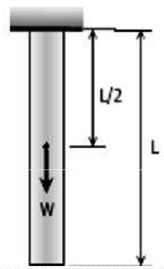
$$\delta = \frac{\rho g}{E}\int_0^L y\, dy = \frac{\rho g}{E}\left[\frac{y^2}{2}\right]_0^L$$

$$\delta = \frac{\rho g}{2E}\left[L^2 - 0^2\right] = \rho g L^2/2E \qquad ok!$$

Given the total mass M:
$\rho = M/V = M/AL$

$\delta = \rho g L^2/2E = (M/AL)(gL^2/2E)$
$\delta = MgL/2AE \qquad ok!$

Another Solution:



The weight will act at the center of gravity of the bar:

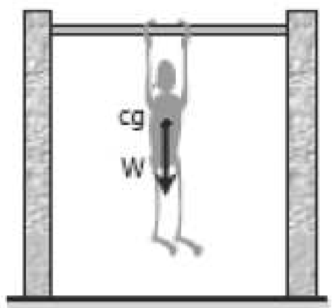
$$\delta = \frac{PL}{AE}$$

Where: $P = W = (\rho A L)g$
$L = L/2$

$$\delta = \frac{[(\rho A L)g](L/2)}{AE}$$

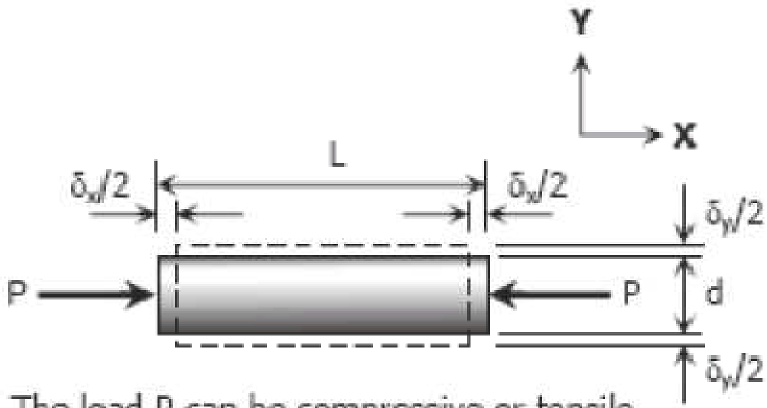
$$\delta = \frac{\rho g L^2}{2E} \qquad ok!$$

For you to feel the situation, position yourself in pull-up exercise with your hands on the bar and your body hang freely above the ground. Notice that your arms suffer all your weight and your lower body fells no stress (center of weight is approximately just below the chest). If your body is the bar, the elongation will occur at the upper half of it.



- **Problem 222** A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is 4Pν / πEd.

Solution 222



The load P can be compressive or tensile

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\varepsilon_x$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

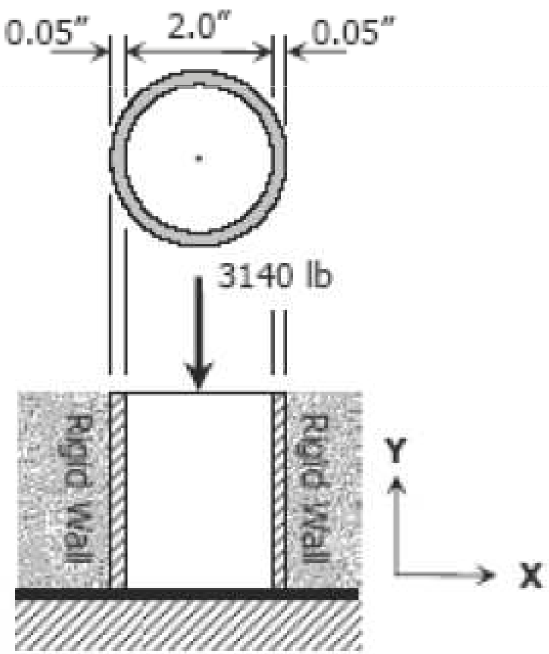
$$\frac{\delta_y}{d} = -\nu\frac{-P}{AE}$$

$$\delta_y = \nu\frac{Pd}{\frac{1}{4}\pi d^2 E}$$

$$\delta_y = \frac{4P\nu}{\pi E d} \qquad ok!$$

Problem 226 A 2-in.-diameter steel tube with a wall thickness of 0.05 inch just fits in a rigid hole. Find the tangential stress if an axial compressive load of 3140 lb is applied. Assume ν = 0.30 and neglect the possibility of buckling.

Solution 226



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\sigma_y$$

where σ_x = tangential stress

σ_y = longitudinal stress

$$\sigma_y = \frac{P_y}{A} = \frac{3140}{\pi(2)(0.05)}$$

$$\sigma_y = 31400/\pi \text{ psi}$$

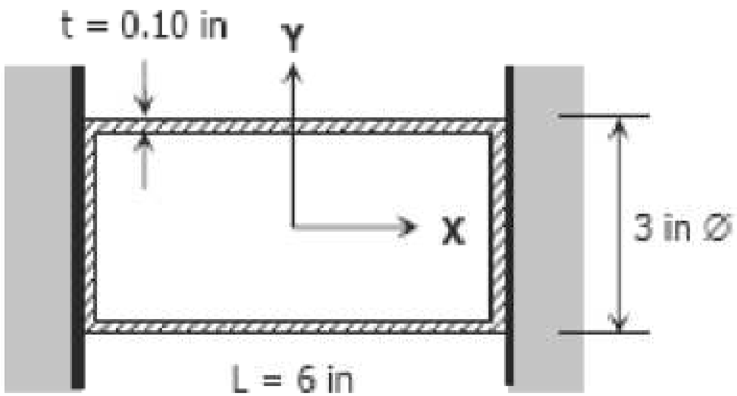
$$\sigma_x = 0.30(31400/\pi)$$

$$\sigma_x = 9430/\pi \text{ psi}$$

$$\sigma_x = 2298.5 \text{ psi}$$

- **Problem 228** A 6-in.-long bronze tube, with closed ends, is 3 in. in diameter with a wall thickness of 0.10 in. With no internal pressure, the tube just fits between two rigid end walls. Calculate the longitudinal and tangential stresses for an internal pressure of 6000 psi. Assume ν = 1/3 and E = 12 × 106 psi.

Solution 228



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$\sigma_x = \nu\sigma_y = \sigma_l$ → longitudinal stress

$\sigma_t = \sigma_y$ → tangential stress

$$\sigma_t = \frac{pD}{2t} = \frac{6000(3)}{2(0.10)}$$

$$\sigma_t = \mathbf{90{,}000\ psi}$$

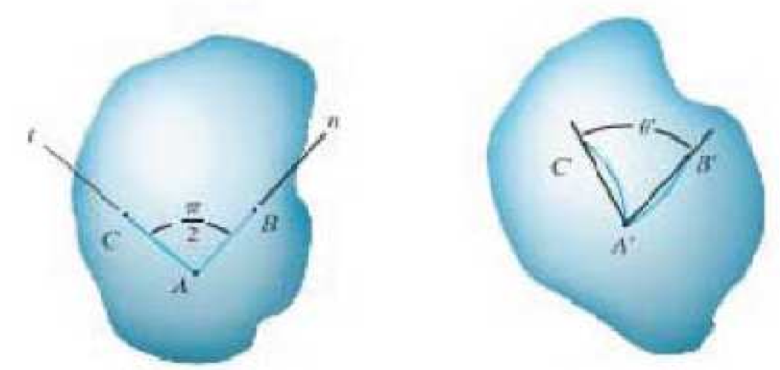
$$\sigma_l = \nu\sigma_y = \tfrac{1}{3}(90{,}000)$$

$$\sigma_l = \mathbf{30{,}000\ psi}$$

Shear Strain

Shear Strain. Deformations not only cause line segments to elongate or contract, but they also cause them to change direction. If we select two line segments that are originally perpendicular to one another, then the change in angle that occurs between these two line segments is referred to as ***shear strain***. This angle is denoted by γ (gamma) and is always measured in radians (rad), which are dimensionless. For example, consider the line segments AB and AC originating from the same point A in a body, and directed along the perpendicular n and t axes, Fig. 2–2a. After deformation, the ends of both lines are displaced, and the lines themselves become curves, such that the angle between them at A is θ', Fig. 2–2b. Hence the shear strain at point A associated with the n and t axes becomes

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$



Undeformed body (a)

Deformed body (b)

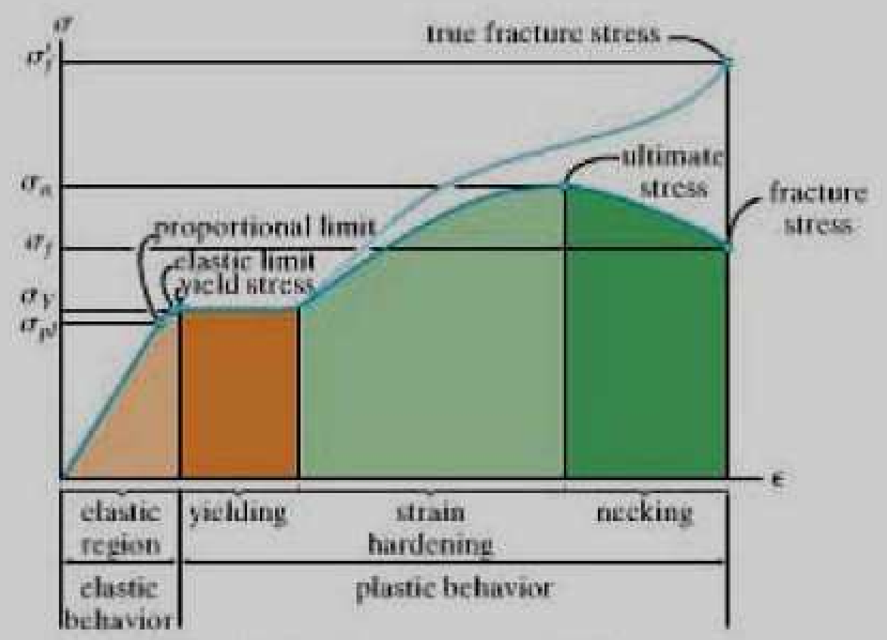
Important Points

- Loads will cause all material bodies to deform and, as a result, points in a body will undergo *displacements or changes in position.*
- *Normal strain* is a measure per unit length of the elongation or contraction of a small line segment in the body, whereas *shear strain* is a measure of the change in angle that occurs between two small line segments that are originally perpendicular to one another.
- The state of strain at a point is characterized by six strain components: three normal strains ϵ_x, ϵ_y, ϵ_z and three shear strains γ_{xy}, γ_{yz}, γ_{xz}. These components depend upon the original orientation of the line segments and their location in the body.
- Strain is the geometrical quantity that is measured using experimental techniques. Once obtained, the stress in the body can then be determined from material property relations, as discussed in the next chapter.
- Most engineering materials undergo very small deformations and so the normal strain $\epsilon \ll 1$. This assumption of "small strain analysis" allows the calculations for normal strain to be simplified, since first-order approximations can be made about their size.

Stress strain diagram of material

Elastic Behavior. Elastic behavior of the material occurs when the strains in the specimen are within the light orange region shown in Fig. 3–4. Here the curve is actually a *straight line* throughout most of this region, so that the stress is *proportional* to the strain. The material in this region is said to be *linear elastic*. The upper stress limit to this linear relationship is called the ***proportional limit***, σ_{pl}. If the stress slightly exceeds the proportional limit, the curve tends to bend and flatten out as shown. This continues until the stress reaches the ***elastic limit***. Upon reaching this point, if the load is removed the specimen will still return back to its original shape. Normally for steel, however, the elastic limit is seldom determined, since it is very close to the proportional limit and therefore rather difficult to detect.

Yielding. A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently*. This behavior is called ***yielding***, and it is indicated by the rectangular dark orange region of the curve. The stress that causes yielding is called the ***yield stress*** or ***yield point***, σ_Y, and the deformation that occurs is called ***plastic deformation***. Although not shown in Fig. 3–4, for low-carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The ***upper yield point*** occurs first, followed by a sudden decrease in load-carrying capacity to a ***lower yield point***. Notice that once the yield point is reached, then as shown in Fig. 3–4, the specimen will continue to elongate (strain) *without any increase in load*. When the material is in this state, it is often referred to as being ***perfectly plastic***.



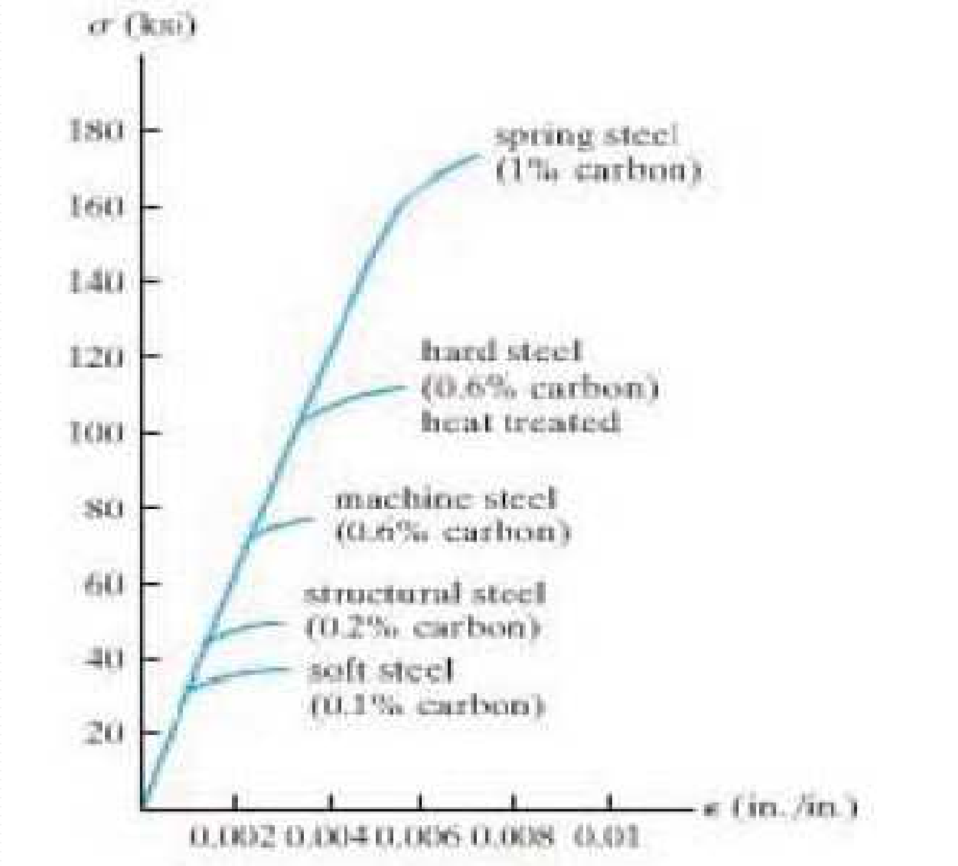
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Strain Hardening. When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ***ultimate stress***, σ_u. The rise in the curve in this manner is called ***strain hardening***, and it is identified in Fig. 3–4 as the region in light green.

Necking. Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is fairly *uniform* over the specimen's entire gauge length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a *localized* region of the specimen. As a result, a constriction or "neck" tends to form in this region as the specimen elongates further, Fig. 3–5*a*. This region of the curve due to necking is indicated in dark green in Fig. 3–4. Here the stress–strain diagram tends to curve downward until the specimen breaks at the ***fracture stress***, σ_f, Fig. 3–5*b*.

Hook's Law

- In most of the material, the stress strain relationship is linear i.e. Increase in stress leads to proportionate increase in the strain:
- E represents Young's Mod $\sigma = E\epsilon$ steel E is give as



$$E = \frac{\sigma_{pl}}{\epsilon_{pl}} = \frac{35 \text{ ksi}}{0.0012 \text{ in./in.}} = 29(10^3) \text{ ksi}$$

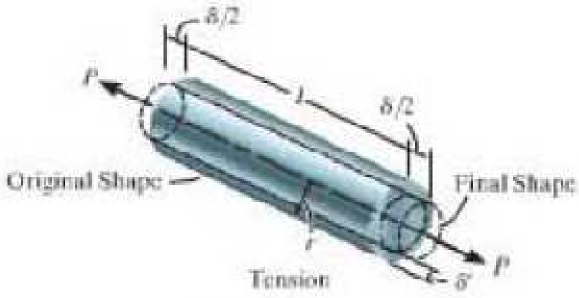
Important Points

- A *conventional stress–strain diagram* is important in engineering since it provides a means for obtaining data about a material's tensile or compressive strength without regard for the material's physical size or shape.
- *Engineering stress and strain* are calculated using the *original* cross-sectional area and gauge length of the specimen.
- A *ductile material*, such as mild steel, has four distinct behaviors as it is loaded. They are *elastic behavior, yielding, strain hardening,* and *necking*.
- A material is *linear elastic* if the stress is proportional to the strain within the elastic region. This behavior is described by *Hooke's law*, $\sigma = E\epsilon$, where the *modulus of elasticity* E is the slope of the line.
- Important points on the stress–strain diagram are the *proportional limit, elastic limit, yield stress, ultimate stress*, and *fracture stress*.
- The *ductility* of a material can be specified by the specimen's *percent elongation* or the *percent reduction in area*.
- If a material does not have a distinct yield point, a *yield strength* can be specified using a graphical procedure such as the *offset method*.
- *Brittle materials*, such as gray cast iron, have very little or no yielding and so they can fracture suddenly.
- *Strain hardening* is used to establish a higher yield point for a material. This is done by straining the material beyond the elastic limit, then releasing the load. The modulus of elasticity remains the same; however, the material's ductility *decreases*.
- *Strain energy* is energy stored in a material due to its deformation. This energy per unit volume is called *strain-energy density*. If it is measured up to the proportional limit, it is referred to as the *modulus of resilience*, and if it is measured up to the point of fracture, it is called the *modulus of toughness*. It can be determined from the area under the $\sigma - \epsilon$ diagram.

3.6 Poisson's Ratio

When a deformable body is subjected to an axial tensile force, not only does it elongate but it also contracts laterally. For example, if a rubber band is stretched, it can be noted that both the thickness and width of the band are decreased. Likewise, a compressive force acting on a body causes it to contract in the direction of the force and yet its sides expand laterally.

Consider a bar having an original radius r and length L and subjected to the tensile force P in Fig. 3–21. This force elongates the bar by an amount δ, and its radius contracts by an amount δ'. Strains in the longitudinal or axial direction and in the lateral or radial direction are, respectively,



$$\epsilon_{long} = \frac{\delta}{L} \quad \text{and} \quad \epsilon_{lat} = \frac{\delta'}{r}$$

In the early 1800s, the French scientist S. D. Poisson realized that within tl *elastic range* the *ratio* of these strains is a *constant*, since the deformatioı δ and δ' are proportional. This constant is referred to as ***Poisson's ratio***, ν (nu), and it has a numerical value that is unique for a particular material that is both *homogeneous and isotropic*. Stated mathematically it is

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}} \tag{3-9}$$

EXAMPLE 3.4

A bar made of A-36 steel has the dimensions shown in Fig. 3–22. If an axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.

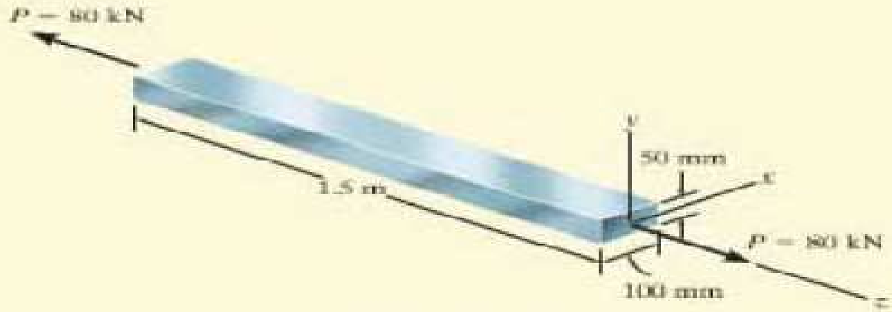


Fig. 3–22

SOLUTION

The normal stress in the bar is

$$\sigma_z = \frac{P}{A} = \frac{80(10^3)\text{ N}}{(0.1\text{ m})(0.05\text{ m})} = 16.0(10^6)\text{ Pa}$$

From the table on the inside back cover for A-36 steel $E_{st} = 200$ GPa, and so the strain in the z direction is

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\text{ Pa}}{200(10^9)\text{ Pa}} = 80(10^{-6})\text{ mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\text{ m}) = 120\ \mu\text{m} \qquad \textit{Ans.}$$

Using Eq. 3–9, where $\nu_{st} = 0.32$ as found from the inside back cover, the lateral contraction strains in *both* the x and y directions are

$$\epsilon_x = \epsilon_y = -\nu_{st}\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are

$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\text{ m}) = -2.56\ \mu\text{m} \qquad \textit{Ans.}$$

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\text{ m}) = -1.28\ \mu\text{m} \qquad \textit{Ans.}$$