1. INTRODUCTION

Shear strength of reinforced concrete (RC) is determined with the help of certain empirical equations based on experimental results from the normal strength reinforced concrete (NSRC) beams.

According to American Concrete Institute Building Code 318 [1], the shear strength of concrete members without transverse reinforcement subject to shear and flexure is given by the following equation:

\[ V_c = (0.16 \sqrt{f'_c} + 17 \rho \frac{V_u d}{M_u}) b_w d \]  

where \( f'_c = 28 \) days compressive strength of concrete, 
\( \rho \) = longitudinal reinforcement in the web, 
\( V_u \) = factored shear force at the section, 
\( d \) = effective depth of beam and \( b_w \) = web thickness.

\( V_c \leq 3.5 b_w d \) and \( \frac{V_u d}{M_u} \leq 1.0 \) in computing \( V_c \) by Equation (1), where \( M_u \) is factored moment occurring simultaneously with \( V_c \) at section considered.
Based on the experimental research on high strength concrete beams, Maphonde and Frantz [2], Sarsam and Al-Musawi [3], and Ahmad et al. [4] have shown that Equation (1) overestimates the effect of the compressive strength of concrete and underestimates the effect of shear span to depth ratio on the shear strength of HSRC beams. Hence, Equation (1) is mostly valid for NSRC beams.

Due to lack of test data on compressive strengths of concrete greater than 70 MPa (≈10,000 psi), the 1989 edition of the ACI code imposed a maximum value of 0.70 MPa (100psi), for use in the calculation of shear strength of concrete beams, joists, and slabs. Exceptions to this limit were permitted in beams and joists when the transverse reinforcement satisfied an increased value for the minimum amount of web reinforcement.

Vecchio and Collins [5] developed the Compression Field Theory (CFT) to study the effect of tensile stresses on the shear strength of RC beams in the cracked region. The nominal shear capacity \( V_n \) of the reinforced concrete section is given as:

\[
V_n = V_s + V_p
\]

\( V_s \) = shear strength provided by the cracked concrete
\( V_p \) = shear strength provided tensile stress in stirrups
\( V_p \) = vertical component of applied pre-stressed tendons

\[
V_n = \beta \sqrt{f_c b_w d_w + A_f f_y \cot \theta} + V_p
\]  

\( \beta \) = concrete tensile stress factor indicating the ability of diagonally cracked concrete to resist shear
\( d_w \approx 0.9 d \) = the minimum web depth
Rameriz and Breen [6] suggested the following model for nominal shear strength of concrete beams without web reinforcement:

\[
V_n = 0.5(V_{cr} - \theta)b_w d
\]

\( V_{cr} \) = shear stress resulting in the first diagonal tension cracking in the concrete
\( \theta \) = crack angle in radians

Gambarova [7] and Dei Poli et al. [8] developed the approach of the truss model, which is based on the assumption that the forces are transferred across the crack by the friction which depends on the crack displacement (slip and crack width). They proposed the following equation for the contribution of web reinforcement in resisting the shear in RC beams:

\[
V_s = \frac{A_f f_y \cot \beta_{cr}}{s}
\]

where \( \beta_{cr} \) = crack inclination
\( d_w \) = inner lever arm
\( s \) = stirrup spacing
Karim et al. [9] proposed the following equation for predication of ultimate shear stress in beams without web reinforcement.

\[
v_c = \frac{V_u}{bd} = 0.4 + \frac{f_y (\frac{d_f}{d} - 3A_f)}{a} (\text{SI Units})
\]

where \( A_f = \frac{\rho}{d} \) for 1.0 < \( \rho \) < 2.5 and 2.5 for \( \rho \) ≥ 2.5

Zararis [10] has proposed the following models for the shear strength of beams without web reinforcement:

\[
V_{cr} = \left[ 1.2 - 0.2\left(\frac{d_f}{d}\right) \right] \left( \frac{f_{ct}}{f_c} \right)^{0.5} f_{ct} b_d
\]

where \( 1.2 - 0.2\left(\frac{d_f}{d}\right) \geq 0.65 \) (\( d \) in meters)

\[
f_{ct} = 0.3(f_{y})^{0.5}
\]

\( c \) is depth of compression zone which is determined by the following quadratic equation:
Attaullah Shah and Saeed Ahmad

\[ \left( \frac{c}{d} \right)^2 + \frac{600(\rho + \rho')}{f'_c} \left( \frac{c}{d} \right) - \frac{600}{f'_c} \left( \rho + \left( \frac{d}{d'} \right)^2 \rho' \right) = 0 \] (SI Units)  

(8)

For beams with shear reinforcement, the steel contribution is added, which is expressed as

\[ V_s = \left[ 0.5 + 0.25 \left( \frac{a}{d} \right) \right] \rho f_{yy} bd \] (SI units)  

(9)

The shear strength of HSRC beams is still determined by using the equations of Normal Strength Reinforced Concrete (NSRC) beams by most of the codes. However, various researchers have proposed certain empirical equations for shear strength of HSRC beams on the basis of test results and mathematical models.

Sarkar et al. [11] worked on reinforced concrete beams with compressive strength ranging from 40 MPa to 110 MPa. They proposed the following two equations for the shear stress of reinforced concrete beams without web reinforcement.

For beams having \( a/d \leq 2 \)

\[ \nu_c = 4.13 \left( f'_c \rho d \right)^{0.66} \] (SI Units)  

(10)

For beams having \( a/d > 2 \)

\[ \nu_c = 3.05 \left( f'_c \rho d \right)^{0.55} \] (SI Units)  

Bazant and Kim [12] proposed a very reliable expression for computing the shear strength of RC beams without transverse reinforcement, which is given as

\[ \nu_{uv} = \zeta \left[ 0.83 \rho^{1/3} f'_c + 206.9 \rho^{5/6} \left( \frac{a}{d} \right)^{-1/2} \right] \] (SI Units)  

(11)

Where \( \zeta = \frac{1}{\sqrt{a + \frac{d}{2d_a}}} \) is a function taking into account the size effect of aggregates, and where \( d_a \) stands for aggregates sizes.

Russo et al. [13] proposed the following expression for shear strength of HSRC concrete beams without transverse reinforcement based on Bazant and Kim’s equation:

\[ \nu_{uv} = \zeta \left[ 0.97 \rho^{0.46} f'^{1/2} + 0.2 \rho^{0.91} f'_{c,l}^{0.38} \rho^2 \left( \frac{a}{d} \right)^{-2.33} \right] \]  

(12)

They further proposed the following expression for the shear strength of HSRC beams with transverse reinforcement using the above expression:

\[ \nu_{uv} = \zeta \left[ 0.97 \rho^{0.46} f'^{1/2} + 0.2 \rho^{0.91} f'_{c,l}^{0.38} \rho^2 \left( \frac{a}{d} \right)^{-2.33} \right] + 1.75 I_b \rho f_{yy} \]  

(13)

The factor \( I_b \) is given by the equation

\[ I_b = \frac{0.97 \rho^{0.46} f'^{1/2}}{0.97 \rho^{0.46} f'^{1/2} + 0.2 \rho^{0.91} f'_{c,l}^{0.38} \rho^2 \left( \frac{a}{d} \right)^{-2.33}} \]  

(14)

To check whether the shear failure is due to beam action or arch action, the author further proposed a critical value as

\[ \left( \frac{a}{d} \right)_c = 0.57 \rho^{0.19} f_{yy}^{1/2} \] (SI units)  

(15)
Hence \( I_{bc} = 0.57 \), which means that for

i). \( a/d < (a/d)_c \), \( I_b < 0.57 \), arch action prevails

ii). \( a/d > (a/d)_c \), \( I_b > 0.57 \), beam action prevails

Cladera and Mari [14,15] proposed the following equations for the shear strength of beams without web reinforcement

\[
V_c = \left[ 0.225 \zeta (100 \rho)^{1.2} f_c^{0.2} \right] b_w d \quad \text{(SI Units)} \tag{16}
\]

For beams with web reinforcement, the shear strength is given as

\[
V_c = \left[ 0.17 \zeta (100 \rho)^{1.2} f_c^{0.20} \tau^{1.3} \right] b_w d \quad \text{(17)}
\]

\[
V_s = \frac{d_s A_{sw}}{s(f_{yc} d \cot \theta)} \quad \text{(18)}
\]

In the present research, seventy high-strength concrete-reinforced (HSRC) beams with and without web shear have been tested under monotonic load at mid span. Based on the test results, two regression equations have been proposed for predicting the shear strength of HSRC beams. The results have been compared with the existing models proposed by different researchers.

2. DETAILS OF MATERIAL AND TEST SAMPLES

To study the behavior of high strength concrete beams in shear, with and without shear reinforcement, seventy beams in two series of thirty-five beams each of size 23cm x 300 cm (9 in x 12 in) were prepared. Seven values of shear span to depth (a/d) ratios were used to study mainly the slender beams (shear span to depth ratio a/d were taken as 3, 3.5, 4, 4.5, 5, 5.5, 6). For each value of a/d, five types of longitudinal steel ratios were used (\( \rho = 0.0033, 0.0075, 0.01, 0.015, 0.02 \)) to study the effect of longitudinal steel ratios on the shear strength of HSRC beams.

For series-I, thirty-five beams were used without transverse reinforcement, whereas in series-II, thirty-five beams having shear reinforcement with #2 bars @ 6" c/c were used, which corresponds to minimum shear reinforcement as per ACI-318 code provisions.

2.1. Material

2.1.1. Reinforcing steel

For main reinforcement, deformed steel bars having nominal yield stress of 414 MPa (60,000 psi) have been used. For shear reinforcement, plain steel bars of yield stress 276 MPa (40,000 psi) were used.

2.1.2. Concrete

The concrete mix design of the beams used in this experimental program has been given in Table 1. Coarse aggregates of size \( \frac{3}{4} \) in (20mm) and fine aggregates conforming to ASTM standards with modulus of fineness as 2.67 were used in the concrete. High range water reducers conforming to ASTM C-494 type F standards was used at 1.70 % by weight of cement to control the water cement ratio and enhance the compressive strength of concrete. The details of beam sizes, main reinforcement, shear reinforcement, and a/d ratios are shown in Table 2 and a typical section of the beams is given in Figure 1. The loading arrangements are shown in Figure 2.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-I Cement</td>
<td>628 kg/m³</td>
</tr>
<tr>
<td>Fine aggregates</td>
<td>484 kg/m³</td>
</tr>
<tr>
<td>Coarse aggregates</td>
<td>1128 kg/m³</td>
</tr>
<tr>
<td>HRWR @ by weight of cement</td>
<td>10.70 kg/m³</td>
</tr>
<tr>
<td>Water @ 0.25 w/c ratio</td>
<td>157 kg/m³</td>
</tr>
<tr>
<td>Average Design Cylinder Compressive strength (28 days) ( f'_c )</td>
<td>50-54 MPa</td>
</tr>
</tbody>
</table>
Figure 1. Typical section of beams without and with stirrups

Figure 2. Typical loading arrangement for testing of beams
Table 2. Details of Beams With and Without Shear Reinforcement

<table>
<thead>
<tr>
<th>Beams without stirrups Series-I</th>
<th>Beams with stirrups Series-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>ρ (%)</td>
</tr>
<tr>
<td>B1-1</td>
<td>0.33</td>
</tr>
<tr>
<td>B1-2</td>
<td>0.33</td>
</tr>
<tr>
<td>B1-3</td>
<td>0.33</td>
</tr>
<tr>
<td>B1-4</td>
<td>0.33</td>
</tr>
<tr>
<td>B1-5</td>
<td>0.33</td>
</tr>
<tr>
<td>B1-6</td>
<td>0.33</td>
</tr>
<tr>
<td>B1-7</td>
<td>0.33</td>
</tr>
<tr>
<td>B2-1</td>
<td>0.73</td>
</tr>
<tr>
<td>B2-2</td>
<td>0.73</td>
</tr>
<tr>
<td>B2-3</td>
<td>0.73</td>
</tr>
<tr>
<td>B2-4</td>
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<tr>
<td>B2-5</td>
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<td>2.0</td>
</tr>
<tr>
<td>B5-7</td>
<td>2.0</td>
</tr>
</tbody>
</table>
3. TESTING AND OBSERVATIONS

The beams were tested under monotonic concentrated load at the mid span. The schematic testing arrangements of the beams is shown in Figure 2. The beams were cured with moist sand in the curing yard as shown in Figure 3.

When the load was applied and increased gradually, flexural cracks appeared in the beams near the mid span of the beams, which were more or less vertical in nature. With further increase of load, inclined shear cracks developed in the beams, which are sometimes called primary shear cracks as well.

Typical cracking in the slender beams without transverse reinforcement leading to the failure involved two branches. The first branch was a slightly inclined shear crack and was typically of the height of the other flexural cracks developed on the surface of beams. The second branch of the crack, also called secondary shear crack, initiated from the tip of the first crack at a relatively flatter angle, splitting the concrete in the compression zone. This crack further extended in the compression zone and finally met the loading point, leading to the collapse of the beam as shown in Figure 4. The nominal shear strength at the initiation of the second branch crack was taken as the shear capacity of the beams. In case of beams without transverse reinforcement, the secondary shear crack formed shortly after the development of the primary shear crack and the shear failure was sudden, as shown in Figure 5.

In the case of beams with transverse reinforcement, the formation of secondary shear crack was not abrupt and beams carried more loads before failure as compared to beams without web reinforcement. In both cases, the shear strength of the beams was taken at the point when secondary shear cracks appeared.
4. REGRESSION MODEL FOR SHEAR STRENGTH OF BEAMS WITHOUT WEB REINFORCEMENT

The shear strength of HSRC beams was investigated as a function of longitudinal steel $\rho$ expressed in percentage (%), shear span to depth ratio $(a/d)$, compressive strength of concrete, and $f'_c$ (Mpa) for beams without shear reinforcement. The Ordinary Least Square (OLS) estimate for the linear regression model is given as

$$V_c = \left[0.026f'_c + 0.507\rho - 0.208(a/d)\right]bd$$

The actual and predicted values of shear stress based on the linear regression model have been compared in Figure 6 and Figure 7.

5. LINEAR REGRESSION MODEL FOR SHEAR STRENGTH OF BEAMS WITH WEB REINFORCEMENT

According to ACI-318, the shear contribution of stirrups in beams with web reinforcement is given as

$$V_s = \frac{A_y f_y}{s}$$

According to ACI-318, the nominal shear strength of RC beams with web reinforcement is the sum of the individual contributions of concrete and steel.

$$V_n = V_c + V_s = (V_c + V_s)bd$$

However, research by Sarsam and Al-Musawi [16] and P. Regan [17] has revealed that the behavior of stirrups in resisting the shear of RC beams is more complicated and irregular. The actual results of 35 beams with web reinforcement have also shown that the increase in shear strength due to stirrups exhibits a non-linear and non-uniform behavior in resisting the shear.

The linear regression model was worked out for the data of 35 beams tested in the research and the following equation was obtained:

$$V_c = \left[0.01f'_c + 0.507\rho - 0.208(a/d) + 4.53\rho v f_y\right]bd$$

The actual and predicted values by the models have been compared in Figure 8 and Figure 9.

6. COMPARISON OF THE PROPOSED MODELS WITH ACI-318 CODE AND OTHER MODELS

6.1. Beams without Shear Reinforcement

The shear strength of beams without shear reinforcement was compared with the ACI equations [1] and other models proposed by Bezant and Kim [12] and G. Russo et al. [13]. The comparison has been given in Figure 10.

6.2. Beams with Shear Reinforcement

The predicted results of shear strength for beams with shear reinforcement have been compared with the results of ACI Equation [1] and model proposed by G. Russo et al. [13]. The comparison has been given in Figure 11.
Figure 6. Comparison of actual and predicted values of shear stress in beams without web reinforcement for $\rho \leq 1\%$
Figure 7. Comparison of actual and predicted values of shear stress in beams without web reinforcement for $p>1\%$
Figure 8. Comparison of actual and predicted values of shear stress in beams with web reinforcement for $\rho \leq 1\%$
Figure 9. Comparison of actual and predicted values of shear stress in beams with web reinforcement for $\rho > 1\%$.
Figure 10. Comparison of actual shear stress of beams having no shear reinforcement with the proposed equation and other models for $\rho \geq 1\%$
Figure 11. Comparison of actual shear stress of beams having shear reinforcement with the proposed equation and other models for $\rho \geq 1\%$
7. CONCLUSION AND RECOMMENDATIONS

From comparison of the actual and predicted values of shear stress of beams, the following results have been observed:

i. For beams without shear reinforcement, the proposed equation is conservatives for $\rho = 0.75\%$ and $1\%$ and un-conservative for $\rho = 0.33\%$. For $\rho = 2\%$, the equation gives closer values to the actual observations.

ii. For beams with shear reinforcement, the proposed regression equation is conservative for $\rho = 0.33\%$, $0.75\%$ and un-conservative for $\rho = 1\%$.

iii. For $\rho = 1.5\%$ and $2\%$, the equation gives closer values to the actual.

From comparison of the predicted values of shear stress and values proposed by ACI, the Russo et al. equation, and the Bazant et al. equation, the following results have been observed:

i. The Bazant et al. equation is un-conservative in estimating the shear stress for the HSRC beams without web reinforcement as it overestimates the shear stress for all values of longitudinal steel.

ii. The Russo et al. equation is more conservative as it underestimates the shear stress of the HSRC beams without web reinforcement.

iii. The ACI-318 equation for shear stress of HSRC beams gives some reasonable values when compared with the actual and predicted values.

iv. The Russo et al. equation, on the other hand, is un-conservative for shear stress of HSRC beams with web reinforcement.

v. The proposed regression equation better estimates the shear stress of beams as compared to the other models of ACI, Russo and Bazant. However, this may be due to the fact that the equations are based on the data of the tests’ results. Hence, for generalization of the regression equation to other sets of tests’ data, more experimental research is required.

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